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Numerical Study of Unsteady Supercavitation and Flow Induced Noise Propagation

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Abstract

Operation of hydraulic devices and underwater vehicles is often accompanied by occurrence of flow cavitation/supercavitation. Due to the transient and unstable nature of cavitation phenomenon, it is difficult to predict the evolution of unsteady cavitation/supercavitation. The current study is focused on the numerical investigation of supercavitating liquid flow perturbed by pressure wave with one-fluid cavitation model. It is observed that the supercavity will become unstable under the impact of pressure wave and may collapse or deform locally, depending on strength of perturbation. Collapse of supercavitation results in huge pressure surge, which may cause the material erosion, noise, vibration and efficiency loss of operating devices. Understanding of unsteady supercavitation is crucial for design and optimization of such devices. Separately, the supercavity inception/development and its sustainment through ventilated cavitation may result in turbulence and fluctuations at the water-vapor interface, which are major sources of hydrodynamic noise. Here, three main sources are investigated: flow generated noise due to turbulent pressure fluctuations around the supercavity, pressure fluctuations at the vapor-water interface and pressure fluctuations due to direct impingement of ventilated gas-jets. In this study, the near and far field noise contributions from each of the aforementioned sound sources are considered via the boundary integral method. BEM based acoustic solver was developed for computing flow generated sound with flow data obtained from the CFD solver for simulating supercavitation flow.

Keywords: supercavitation, BEM, acoustic perturbation
1 Introduction

Flow supercavitation occurs when fluid is accelerated over a sharp edge, usually at the nose of an underwater vehicle, where phase change occurs and a low density gaseous cavity gradually envelops the whole object (supercavity). This enables higher speeds of underwater vehicles. There are a lot of physics underlying supercavitation, which include presence of interface separating different phases, large constituent density and pressure ratio across phase interface, significant heat and mass transfer, possible non-equilibrium effects with phase transition, large change in sound speed in different flow regions and in resulting flow regime, etc. These physical effects require special attention in numerical modeling and therefore numerical simulation and analysis of cavitating flow is quite difficult. In this study, the unsteady supercavitating flow is resolved in the framework of a compressible homogeneous mixture model, where the compressibility effects of liquid water are taken into account. Moreover, the process of supercavity inception/development by means of ‘natural cavitation’ results in turbulence and fluctuations at the water-vapour interface, which are major sources of hydrodynamic noise. Although flow noise caused by turbulent fluctuations usually dominate the acoustic field, they manifest as ‘pseudo noise’ which can be minimized for efficient operation of acoustic sensors situated within the cavitation field. Besides flow noise, it is important to study the effects of ventilated gas jet impingement and incoming acoustic perturbations on the upstream acoustic field near a cavitation [1]. The present study considers the aforementioned sound sources via the boundary integral method and a BEM based acoustic solver that is devoid of the non-uniqueness problem has been developed. Further, the effect of subsonic motion on the acoustic field has also been studied over a range of frequencies.

The manuscript has been divided into 4 major sections including the introductory section. Section 2 describes unsteady supercavitation. Section 3 considers the effects of ventilated gas jet impingement and an incoming acoustic perturbation on noise propagation around a subsonically moving supercavity. Finally, Section 4 presents the conclusions of the study.

2 Unsteady supercavitation

2.1 Physical model and numerical method

The physical model employed here is constructed by assuming that liquid and vapor phases remain in the kinematic and thermodynamic equilibrium, that is, the different constituents contained in a fluid element share the same velocity, pressure and temperature and the cavitating flow is treated as a single-fluid flow. The governing equations in Cartesian coordinate system is the compressible Euler equations,

\[ \frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = -\frac{i}{y} \mathbf{S} \]  \hspace{1cm} (1)

where \( i \) takes value of 0 and 1 for planar and axisymmetric flows, respectively. Here, \( \mathbf{U}, \mathbf{F}, \mathbf{G} \) and \( \mathbf{S} \) are the vector of conserved variables, inviscid flux vectors in \( x \) and \( y \) directions, geometric source term vector associated with axisymmetric flow, respectively, and are given by,
In expression (2), \( \rho \) is the averaged density, \( u \) the averaged \( x \) velocity component, \( v \) the averaged \( y \) velocity component and \( p \) the averaged pressure. Energy equation is decoupled from model system (1) as the flow is assumed to be isentropic, i.e. the pressure is the function of density only.

In this study, thermodynamic properties of liquid are modeled by Tait EOS,

\[
p = B \left( \frac{\rho}{\rho_0} \right)^N - B + A, \quad p \geq p_{sw}\]

In the above equation, \( N, A, B, \rho_0 \) are the material-dependent constants. For liquid water, they are set to be \( N = 7.15 \), \( A = 10^6 \) Pa, \( B = 3.31 \times 10^8 \) Pa, \( \rho_0 = 1000 \) Kg/m\(^3\), respectively. Here, \( p_{sat} \) is the saturated vapour pressure.

When local flow pressure drops below the saturated vapour condition, water is vaporized, generating cavitation bubble which is filled with the mixture of liquid and vapor and modeled by isentropic formulation [2],

\[
\rho = \frac{k \rho_{g}^{cav} + \rho_{l}^{cav}}{\left( \frac{p}{p_{cav}} + B - A \right)^{1/N}} + k \left( \frac{p}{p_{cav}} \right)^{-1/N}, \quad p \leq p_{sw}
\]

Here, \( \rho_{g}^{cav} \) and \( \rho_{l}^{cav} \) are the associated vapour and liquid densities at the cavitation pressure \( p_{cav} \). Another important physical quantity is speed of sound and given by,

\[
a = \left\{ \rho \left[ \frac{\alpha}{\rho_{sv}, a_{sv}^2} + \frac{(1-\alpha)}{\rho_{sw}, a_{sw}^2} \right] \right\}^{1/2}
\]

where \( a_{sv} \) and \( a_{sw} \) are the speeds of sound of vapour and liquid at a given pressure, respectively. The void fraction \( \alpha \) comes from the mixture density definition, \( \rho = \alpha \rho_{sv} + (1-\alpha) \rho_{sw} \). Here, \( \rho_{sv} \) and \( \rho_{sw} \) are the saturated vapor and water densities, respectively.

Spatial terms of governing equations (1) are discretized on quadrilateral mesh using finite volume MUSCL scheme. The time-marching is handled with second-order Runge-Kutta method.
Special attention is paid to the treatment of stiffness of Tait EOS and solid boundary condition [2].

2.2 Simulation of 2D axisymmetric supercavitation

Numerical simulation is conducted to revolve the two-dimensional axisymmetric supercavitation around a flat-nose cylinder of radius 10mm and length 150mm. The freestream pressure and temperature are 1atm and 300K, respectively. Water enters the computational domain from the left side with the fixed velocity of 45m/s and exits the outlet boundary with a static pressure specified. A quadrilateral mesh of 12,4000-cell is used in the simulation below.

The predicted flow is essentially transient. The flow field features a supercavitation bubble containing the whole cylinder and a re-entrant jet in wake that propagates upstream but is halted by the right end of cylinder. Next, a pressure wave is introduced by increasing the freestream velocity at inlet abruptly from 45m/s to 55m/s. The resulting unsteady evolution of supercavitating flow is shown in Figure 1. It is seen that with the pressure wave propagating downstream, the supercavity collapses from its leading edge near the cylinder nose, shrinks and eventually vanishes. The supercavitation appears to be sensitive to external perturbation.

![Figure 1: Density contours showing the evolution of supercavity with the incoming flow velocity abruptly increased from 45m/s to 55m/s.](image-url)
3 Acoustic study of supercavitation noise

3.1 Boundary integral formulation

The convective wave equation in an acoustic medium with sound speed $c$ and sound pressure $p$ as state variable is given by

$$\nabla^2 p + k_0^2 p - 2i k_0 M \frac{\partial p}{\partial z} - M^2 \frac{\partial^2 p}{\partial z^2} = p_{inc}$$  \hspace{1cm} (6)

where $M$ denotes the Mach number ($M = V_s / c$) of the moving surface along the $z$ direction and $k_0$ denotes the wavenumber of the acoustic wave. The incident wave field is denoted by $p_{inc}$. When $V_s = 0$, Eq. (6) reduces to the conventional Helmholtz equation. By adopting the Prandtl-Glauret transformation, the convected wave equation can be associated with the standard Helmholtz equation for a stationary problem. The transformed Cartesian coordinates are then given by

$$\tilde{x} = x; \tilde{y} = y; \tilde{z} = \frac{z}{\beta}$$  \hspace{1cm} (7)

where $\beta = \sqrt{1 - M^2}$. Equation (6) can be expressed as conventional Helmholtz equation in transformed domain by introducing the transformation $\tilde{p} = p e^{-i\kappa z}$ for the dependent acoustic variable $p$ (where $\kappa = k_0 M / \beta$). The transformed Helmholtz equation becomes

$$\nabla^2 \tilde{p} + \kappa^2 \tilde{p} = \tilde{p}_{inc}$$  \hspace{1cm} (8)

Equation (8) can be represented using the standard boundary integral equation for a scattering problem and is not presented here for the sake of brevity. It should be noted that the boundary conditions of the actual problem are introduced as transformed boundary conditions in Equation (8). In order to ensure that the boundary integral equation yields correct values at the fictitious frequencies of the exterior problem, we incorporate the Burton and Miller method. Owing to symmetry of the supercavity, we use an axisymmetric boundary integral model and use constant elements to represent the boundary acoustic variables. The solution obtained by solving Eq. (8) is then transformed back to the original domain to yield the unknown acoustic variables. The details of the formulation are given in reference [3]. The accuracy of the boundary element code has been verified elsewhere [4] and is not presented here for the sake of brevity.
3.2 Acoustic sources on supercavitating projectile

3.2.1 Ventilated gas jet impingement on supercavity

In this section, we study the effect of ventilated gas jet impingement on the supercavity wall. Figure 2 describes the boundary conditions considered for the present case. The values of jet impact pressure are obtained from the results of Strong et al. [5] who studied the acoustic pressure exerted by gas jets impinging on a plane rigid wall. Figure 3 presents the distribution of sound pressure levels for low and high frequencies. The effect of supercavity motion is clearly observed for $k_0a = 8$, where the sound pressure is convected along the supercavity with increasing Mach numbers. The enlarged views of the cavitator region at $k_0a = 8$ indicate the existence of wave interference which increases with Mach number.

3.2.2 Incoming acoustic perturbation on supercavity

In this section, we study the effect of an incident acoustic perturbation on the acoustic field around a supercavity. A plane wave of unit amplitude is incident on a supercavity at $0^\circ$ (see Fig. 2). The boundary conditions are assumed to be the same as that for the earlier case. Figure 4 presents the total acoustic field resulting from an incident acoustic perturbation at various Mach numbers. The contours indicate acoustic interference with increase in Mach number.

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**Figure 2: Layout showing acoustic sources and boundary conditions**
$k_0a$, $M = 0$, $M = 0.3$, $M = 0.5$

Enlarged views of acoustic field near cavitator region

Figure 3: Sound pressure level distribution at $k_0a = 0.5$ and $k_0a = 8$ for various Mach numbers
### Figure 4: Total pressure distribution at $k_0a = 0.5$, 3 and 8 for various Mach numbers

<table>
<thead>
<tr>
<th>$k_0a$</th>
<th>$M = 0$</th>
<th>$M = 0.3$</th>
<th>$M = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
<tr>
<td>3</td>
<td><img src="image4.png" alt="Image" /></td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
<tr>
<td>8</td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
</tr>
</tbody>
</table>

### Figure 5: Scattered sound pressure level directivity for various Mach numbers at a radius of 10a around the supercavity

<table>
<thead>
<tr>
<th>$k_0a$</th>
<th>$M = 0$</th>
<th>$M = 0.3$</th>
<th>$M = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
</tr>
<tr>
<td>8</td>
<td><img src="image13.png" alt="Image" /></td>
<td><img src="image14.png" alt="Image" /></td>
<td><img src="image15.png" alt="Image" /></td>
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</tbody>
</table>

Legend:  
- **Red** $M = 0$  
- **Black** $M = 0.3$  
- **Blue dashed** $M = 0.5$
As expected, the interference patterns are quite prominent for higher frequencies compared to low frequency. The magnitude of sound pressure is higher in the vicinity of the rigid nose section. Referring to the sound directivity pattern in Figure 5, it can be said that the scattered sound pressure level (SPL) in the vicinity of nose section at low frequency is less by 10 dB compared to high frequency. Furthermore an increase in Mach number for low frequency reduces the sound pressure level by 5 dB near the nose section and the distribution is characterised by relatively low pressure fluctuations around the supercavity. On the other hand, the SPL distribution for high frequency is marked with fluctuations for increasing Mach numbers and shows minimal change in magnitude around the nose section. Figure 6 presents the far field SPL directivity at a distance of 50a around the supercavity. The SPL decreases by 20 dB around the leading edge as compared to the corresponding near field magnitude. As expected, the magnitude of SPL for high frequency at the far field trailing edge, decays relatively faster compared to the low frequency.

4 Conclusions
The unsteady supercavitation around a flat-nose cylindrical body submerged in a water flow is simulated with a homogeneous one-fluid cavitation model. It is observed that the supercavity collapses under the impact of introduced pressure wave. The present study also evaluates the effects of ventilated gas jet impingement and incoming acoustic perturbation on a subsonically moving supercavity. The boundary element solver developed to study the aforesaid problem considers the motion of the supercavity and uses a transformed boundary integral equation to solve for unknown acoustic variables. Hence, the use of transformed boundary integral equation enables the representation of the convective Helmholtz equation in the standard form and easily incorporates the Burton and Miller method to circumvent irregular frequencies. The results of
acoustic study reveal that the effect of subsonic motions on the acoustic pressure at the nose for the case of ventilated gas jet impingement is less compared to the effect of an incident acoustic perturbation. The supercavity is rather sensitive to incoming acoustic perturbation and the effect may be magnified with increase in Mach numbers. The fluctuations in SPL directivity for the high frequency incoming acoustic perturbation indicate that the performance of acoustic sensors located in the cavitator section may deteriorate resulting in low signal to noise ratios. This effect can however be minimised by employing a compliant nose section. Therefore the effect of nose section material and nose shape on the scattered near field will be studied in future.

References


