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Evolution modelling of fast and slow magnetoacoustic waves in thermally unstable plasma

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Abstract

The nonlinear evolution of fast and slow magnetoacoustic waves in the plasma medium with non-adiabatic heating/cooling processes is under consideration. We assume that the magnetic field vector is inclined at an arbitrary angle to the direction of magnetoacoustic wave propagation. The non-adiabatic processes depend on temperature and density and result in the steady non-equilibrium state of the medium. The steady state caused by the balance between heating and cooling rates gives possibility for various thermal instabilities to appear. In current paper, we discuss the wave mode of thermal instability (so–called isentropic instability) and neglect the presence of other modes of thermal instability. The isentropic mode influences on acoustic and magnetoacoustic waves and causes wave amplification. The linear analysis predicts simultaneous amplification of fast and slow magnetoacoustic waves with different increments. Furthermore, our analysis predicts simultaneous disintegration of fast and slow waves on the sequences of autowave (self-sustaining) shock pulses. These results are proved by the numerical simulation of full system of one-dimensional magneto-hydrodynamic equations. The simulation is conducted using the implicit fully conservative difference scheme. The results of numerical modeling show the disintegration of initial perturbation on the sequence of fast and slow shock pulses. The parameters of obtained autowave pulses are in good agreement with values predicted by our analytical model.

Keywords: acoustic instability, magnetoacoustic wave, self-sustained structures

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1 Introduction

The work to be presented describes some important features of magnetoacoustic wave in thermally unstable medium which were not described in details in our previous papers. To be more precise, we will show how the strength of external magnetic field effects on the amplification of fast and slow magnetoacoustic waves.

However, to introduce the topic, we should clarify the main aspects of the problem under consideration. The considered instability of the medium is a consequence of temperature and density dependent non-adiabatic processes in it. The nature of these processes is irrelevant; the most significant part is the specific dependence on the thermodynamic parameters of the medium. There are three possible types of this dependence which results in qualitatively different consequences. These types are known as isochoric, isobaric and isentropic instabilities [1]. The most interesting for us is the isentropic instability because it causes amplification of acoustic modes. It should be mentioned that, amplification of acoustic modes depends on it frequency (i.e. wave increment is frequency dependent). Moreover, the phase and group velocity of acoustic modes are also frequency dependent. These features make thermally unstable medium very similar to the non-equilibrium media. Particularly, similar influence on the wave properties was shown for vibrational excited gas [2]. The comprehensive nonlinear analysis of the wave dynamics in thermally unstable gaseous medium and vibrational excited gas can be found in [2-4].

The mentioned above features take place not only in gaseous media but also in plasma [5, 6]. The plasma medium in straight external magnetic field gives possibility for realization of various non-linear processes by itself. Therefore, the wave dynamics in thermally unstable plasma seems to be very interesting field of research.

This paper is subdivided in four Sections. In Section 2 we will describe our mathematical model and made assumptions. Further, in Section 3 we will show how the strength of external magnetic field effects on the amplification of fast and slow magnetoacoustic waves. Finally, we will show results of our numerical simulation in Section 4.

2 Model and assumptions

The magnetoacoustic waves are studied in uniform fully-ionized plasma for which a Cartesian *x, y, z* coordinate system is adopted. The equilibrium state of the medium is described by the density ρ_0, temperature T_0, pressure P_0 and equilibrium magnetic field vector $\vec{B}_0$, which lies in *x-z* plane. The magnetic field vector has magnitude B_0 and angle θ with respect to *z*-axis. We analyze one-dimensional dynamic of the waves and assume that waves propagate along *z*-axis. During the course of current investigation we neglect the influence of all possible dissipative processes. The non-adiabatic processes are taken into account by the use of heat-loss function $\Im(\rho,T)=L(\rho,T)-Q(\rho,T)$, here L and Q are cooling and heating rates, correspondingly. In equilibrium state this function equals to zero $\Im(\rho_0,T_0)=0$

With the foregoing as background, the basic system of equation can be written in the following vector form:

$$\frac{\partial \vec{B}}{\partial t}=rot\left[\vec{V}\times\vec{B}\right];\quad div\vec{B}=0;$$
$$\frac{\partial\rho}{\partial t}+div\rho\vec{V}=0;\qquad \rho\frac{d\vec{V}}{dt}=-\nabla P-\frac{1}{4\pi}\vec{B}\times rot\left[\vec{B}\right]; \tag{1}$$
$$C_{V\infty}\frac{dT}{dt}-\frac{k_B\cdot T}{m\rho}\cdot\frac{d\rho}{dt}=-\Im(\rho,T);\qquad P=\frac{k_B\cdot T\cdot\rho}{m};$$

Here P, ρ, T, $\vec{V}$, $\vec{B}$ are plasma pressure, density, temperature, velocity and magnetic field; k_B denotes Boltzmann constant; $C_{V\infty}$ denotes specific heat at constant volume and m denotes mean particle mass. The system of equation (1) is written in CGS units using substantial derivative $d/dt=\partial/\partial t+\vec{V}\nabla$.

3 Linear analysis

During the linear analysis of the basic equation (1) we use standard methods of perturbation theory. In other words, we expand plasma quantities around their equilibrium value as $f(z,t)=f_0(z,t)+f_1(z,t)$, where f_1 denotes the perturbed quantity and $f_1/f_0\sim\varepsilon<<1$. The analysis of perturbations allows us to describe dispersion properties of fast and slow magnetoacoustic(MA) waves and Alfven waves.

The Alfven waves are not of interest in this paper. It is worth mentioning here that linear Alfven waves are not susceptible to the influence of non-adiabatic processes and propagate along the direction of wave vector with phase speed $c_a=\sqrt{B_0^2/4\pi\rho_0}$. It also should be mentioned that non-adiabatic processes can influence on the Alfven waves in course of the process of non-linear interaction between Alfven waves and acoustic modes [7-9].

The dispersion equation for fast and slow magnetoacoustic waves (2) can be obtained by setting $f_1=\tilde{f}_1\exp(-i\omega t+ikz)$. Here $\tilde{f}_1$ denotes perturbation magnitude and ω,k denotes wave frequency and wavenumber, respectively.

$$\left(\frac{\omega^2}{k^2}\right)^2 - \frac{\omega^2}{k^2}\left\langle c_a^2 + \frac{(C_{P0} - i\omega\tau_0 C_{P\infty})}{(C_{V0} - i\omega\tau_0 C_{V\infty})}\frac{k_Б T_0}{m}\right\rangle + \frac{(C_{P0} - i\omega\tau_0 C_{P\infty})}{(C_{V0} - i\omega\tau_0 C_{V\infty})}\frac{k_Б T_0}{m} c_a^2 \cos^2\theta = 0 \quad (2)$$

In equation (2) the quantity $C_{P\infty}$ denotes specific heat at constant pressure; C_{V0}, C_{P0} denote low-frequency at constant volume and pressure, correspondingly; τ_0 denotes characteristic heating/cooling time. The quantities C_{V0}, C_{P0} have been introduced by analogy with equilibrium specific heats in the vibrational excited gas. However, they have their own physical interpretation [3]. The characteristic heating/cooling time, in turn, subdivides the whole frequency spectrum into the high-frequency $(\omega\tau_0 >> 1)$ and low-frequency $(\omega\tau_0 << 1)$ ranges.

$$C_{P\infty} = C_{V\infty} + \frac{k_Б}{m},\ C_{V0} = \Im_{0T}\tau_0,\ C_{P0} = \tau_0\left(\Im_{0T}T_0 - \Im_{0\rho}\rho_0\right)/T_0,$$
$$\Im_{0T} = \left(\frac{\partial \Im}{\partial T}\right)_{\rho_0,T_0},\ \Im_{0\rho} = \left(\frac{\partial \Im}{\partial \rho}\right)_{\rho_0,T_0},\ \tau_0 = \frac{k_Б T_0}{mQ(\rho_0,T_0)} = \frac{k_Б T_0}{mL(\rho_0,T_0)} \quad (3)$$

Using equation (2) we can easily derive the frequency dependence of the phase speed for slow (s) and fast (f) magnetoacoustic waves:

$$c_{f,s}(\omega) = \frac{\omega}{\mathrm{Re}(k)} = \sqrt{\frac{\left(c_a^2 + c_{Snd}^2\right) \pm \sqrt{\left(c_a^2 + c_{Snd}^2\right)^2 - 4c_{Snd}^2 c_a^2 \cos^2\theta}}{2}}$$
$$c_{Snd}(\omega) = \sqrt{\frac{\left(C_{P0}^2 + \omega^2\tau_0^2 C_{P\infty}^2\right)}{\left(C_{V0}^2 + \omega^2\tau_0^2 C_{V\infty}^2\right)}\frac{k_Б T_0}{m}}\,. \quad (4)$$

Expressions (4) are found by implying that amplification is weak along the wavelength i.e. $\mathrm{Re}(k) >> \mathrm{Im}(k)$. Investigation of expressions (4) shows that in high-frequency range $(\omega\tau_0 >> 1)$ the phase velocity is described by well-known speed $c_{\infty f,s}$ (5), On the contrary, the phase velocity $c_{0f,s}$ in low-frequency range $(\omega\tau_0 << 1)$ is determined by non-adiabatic processes (5).

$$c_{\infty f,s}=\sqrt{0.5\cdot\left(c_a^2+c_\infty^2\pm\sqrt{\left(c_a^2+c_\infty^2\right)^2-4c_\infty^2c_a^2\cos^2\theta}\right)}$$
$$c_{0f,s}=\sqrt{0.5\cdot\left(c_a^2+c_0^2\pm\sqrt{\left(c_a^2+c_0^2\right)^2-4c_0^2c_a^2\cos^2\theta}\right)}, \qquad (5)$$
$$c_0=\sqrt{\frac{C_{P0}}{C_{V0}}\frac{k_BT_0}{m}}=\sqrt{\gamma_0\frac{k_BT_0}{m}},\qquad c_\infty=\sqrt{\frac{C_{P\infty}}{C_{V\infty}}\frac{k_BT_0}{m}}=\sqrt{\gamma_\infty\frac{k_BT_0}{m}}$$

The non-adiabatic processes influence both on the real and imaginary part of the wavenumber k. Thus, the wave increment $\mathrm{Im}(k)$ becomes frequency dependent. For convenience, we show here only its value in high- and low-frequency limits. Here we use low-frequency coefficient of bulk viscosity ξ_0.

$$\alpha_{0f,s}=\mathrm{Im}k_{(\omega\tau_0<<1)}=\frac{\omega^2\xi_0}{2\rho_0c_{0f,s}^3}\frac{c_{0f,s}^2\left(c_{0f,s}^2-c_a^2\right)}{c_0^2\left(2c_{0f,s}^2-\left[c_a^2+c_0^2\right]\right)},$$
$$\alpha_{\infty f,s}=\mathrm{Im}k_{(\omega\tau_0>>1)}=\frac{\xi_0C_{V0}^2}{2\rho_0c_{\infty f,s}^3\tau_0^2C_{V\infty}^2}\frac{c_{\infty f,s}^2\left(c_{\infty f,s}^2-c_a^2\right)}{c_\infty^2\left(2c_{\infty f,s}^2-\left[c_a^2+c_\infty^2\right]\right)}, \qquad (6)$$
$$\xi_0=\frac{\tau_0\rho_0C_{V\infty}(c_\infty^2-c_0^2)}{C_{V0}}$$

In case of infinitely small external magnetic field these expressions can be reduced to the previously obtained increments for gaseous medium.

$$\alpha_{0(acoustic)}=\frac{\omega^2\xi_0}{2\rho_0c_0^3},\qquad \alpha_{\infty(acoustic)}=\frac{\xi_0C_{V0}^2}{2\rho_0c_\infty^3\tau_0^2C_{V\infty}^2} \qquad (7)$$

During the propagation through the plasma the magnitude of the high-frequency magnetoacoustic perturbation will be amplified by a factor of $\exp\{\alpha_{\infty f,s}c_{\infty f,sl}\Delta t\}$ in a time interval Δt. Similarly, during the propagation through the gaseous medium the magnitude of the high-frequency acoustic perturbation will be amplified by a factor of $\exp\{\alpha_\infty c_\infty\Delta t\}$ in a time interval Δt. It seems useful to compare these amplifications. In Figure 1, one can see the angle dependence of normalized increment of fast and slow MA waves for three specific cases $\beta=2,\ \beta=2/\gamma_\infty,\beta=0.5$ ($c_\infty>c_a,c_\infty=c_a,c_\infty<c_a$ correspondingly). The amplification of the fast MA wave is stronger than amplification of the slow MA wave in the medium where pressure

dominates magnetic tension $(c_\infty > c_a)$. The opposite situation occurs in the medium where magnetic tension dominates pressure $(c_\infty < c_a)$. In case of $\beta = 2/\gamma_\infty$ the amplification of both waves is the same and equals to half amplification of pure acoustic waves $\alpha_{\infty f,s} c_{\infty f,s} = 0.5\alpha_\infty c_\infty$

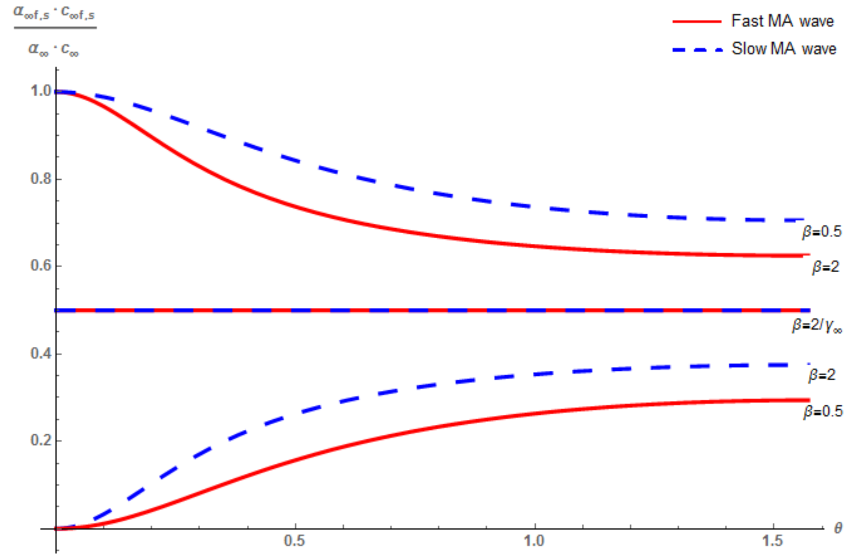


Figure 1: Amplification of fast and slow magnetoacoustic waves scaled by amplification of pure acoustic wave in a time interval Δt

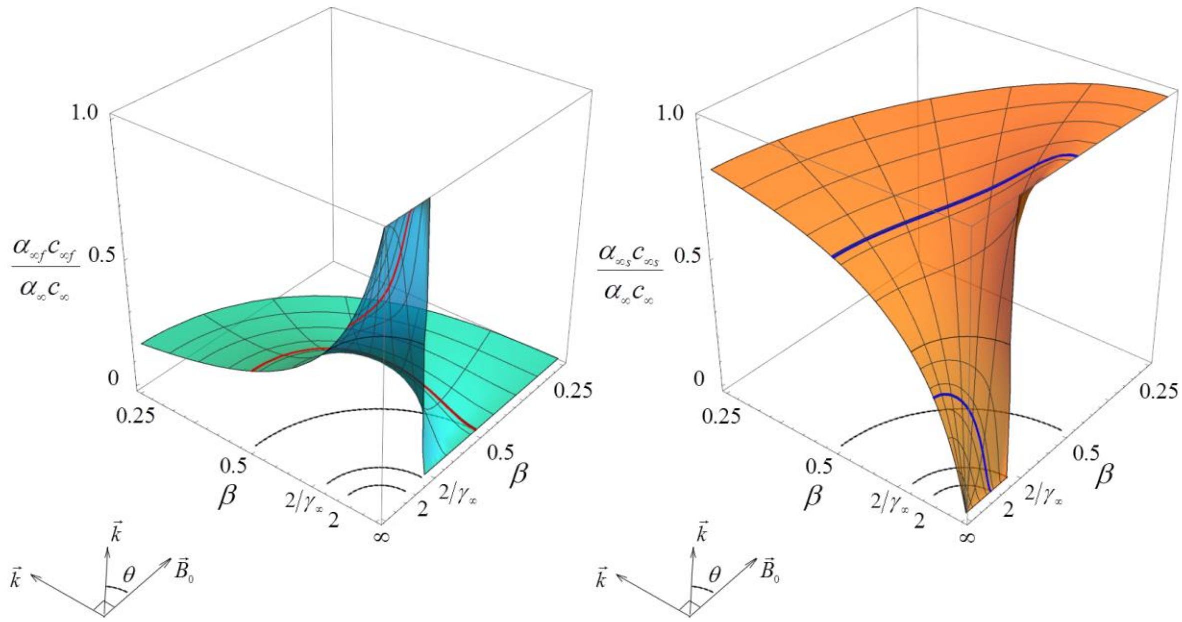


Figure 2: Surfaces of revolution of normalized fast wave increment (left plot) and slow wave increment (right plot) at a radius β. Red and blue lines correspond to the same lines in Figure 1.

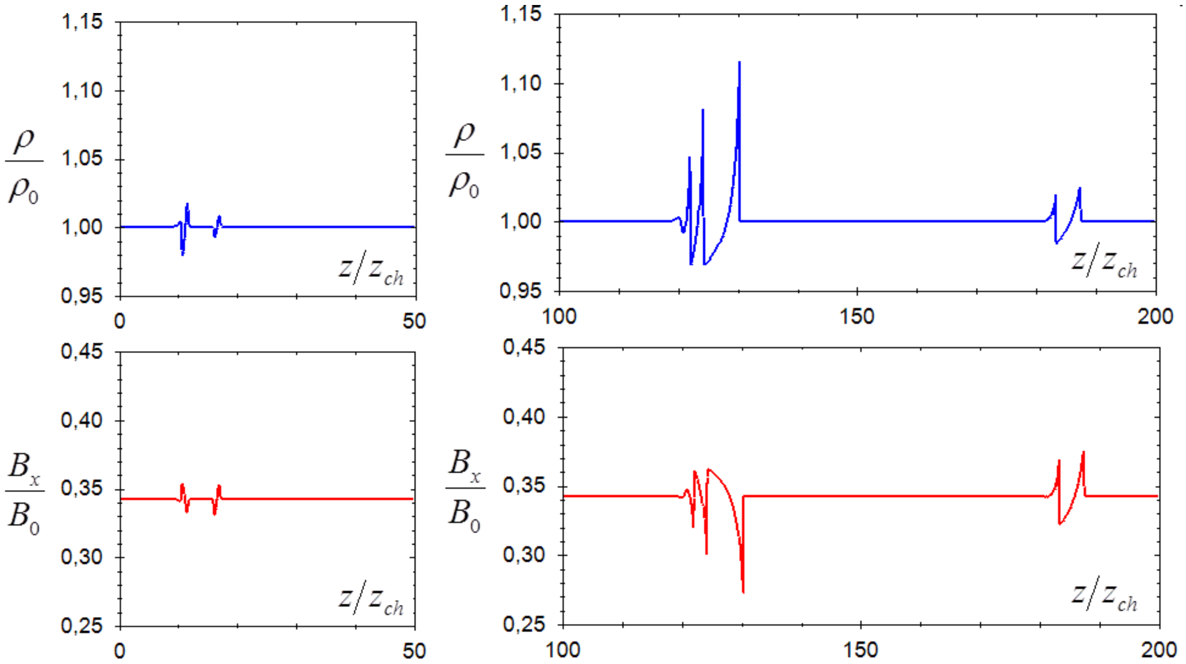
The dependence of fast and slow MA wave amplification on strength of external magnetic field and angle between the wave vector and magnetic field vector can be understood by the plots in Figure 2. To aid the visualization, the presented plots are confined to the interval of plasma beta between infinity (no magnetic field) and 0.25. It seems useful give a special consideration to the cases of propagation along and across direction of magnetic field.

Attention will initially be focused on the case of the wave propagation along the magnetic field. In a medium with dominating pressure $(c_\infty > c_a)$, the high-frequency fast MA wave propagates with the speed of sound c_∞ and will be amplified with the increment of pure acoustic waves $\alpha_{\infty(acoustic)}$. In same conditions, the slow MA wave propagates with the phase speed c_a and have no amplification. For the medium with the dominating magnetic tension $(c_\infty < c_a)$, the situation will be opposite.

In other case, where the direction of wave propagation is across the external magnetic field, only the fast magnetoacoustic wave with the phase speed $c_{\infty f} = \sqrt{c_a^2 + c_\infty^2}$ propagates. The phase speed of the slow magnetoacoustic wave equals 0. It can be easily seen that the growth of the magnetic field magnitude (decrease of plasma beta) causes the decrease of fast wave amplification.

By way of summary, we should notice that for all angles between the wave vector and the magnetic field vector the fast MA waves will be amplified stronger than slow MA waves in plasma with the dominating pressure $\beta > 2/\gamma_\infty$ $(c_\infty > c_a)$. In plasma with the dominating magnetic tension $\beta < 2/\gamma_\infty$ $(c_\infty < c_a)$ the situation is opposite.

It seems useful to support the mentioned above conclusions by the results of numerical simulation. The numerical solving of equations (1) (in Lagrangian hydrodynamic coordinates) was conducted by the use of the fully conservative implicit scheme and the artificial viscosity to smear shocks. The used artificial viscosity is hundred times smaller than the modeled bulk viscosity ξ_0. We also use characteristic spatial scale $z_{ch} = 5c_T\tau_0$ and temporal scale $t_{ch} = 5\tau_0$. In order to show that the amplification of slow waves is stronger than fast wave amplification in plasma with the dominating magnetic tension $\beta < 2/\gamma_\infty$ we use $\beta = 0.85$ and choose the angle between the wave vector and magnetic field $\theta = 20^\circ$. In our simulation we use an initial perturbation of longitudinal velocity V_z in a form of the Gaussian function derivative. Such initial condition results in two pressure pulses which propagate in the opposite direction. During the evolution both of these perturbations decompose on fast and slow pulses. We focused our attention on the fast and slow pulses propagating in positive direction of *z*-axis. These pulses in the moment $t = 10t_{ch}$ are shown in Figure 3a. During the evolution these perturbations decompose again on the sequence of fast and slow magnetoacoustic self-sustained shock pulses. These self-sustained pulses are purely non-linear structures. The compressive description of them can be found in papers [4, 5]. It is worth mentioning here that, governing parameter which specifies pulse magnitudes is wave increment. In this paper, we restrict ourselves on the initial stage of self-sustained structure formation and do not present completely formed shock pulses. In Figure 3b one can see the sequence of the amplified fast pulses and amplified slow pulses with higher magnitude in the moment $t = 110 t_{ch}$.



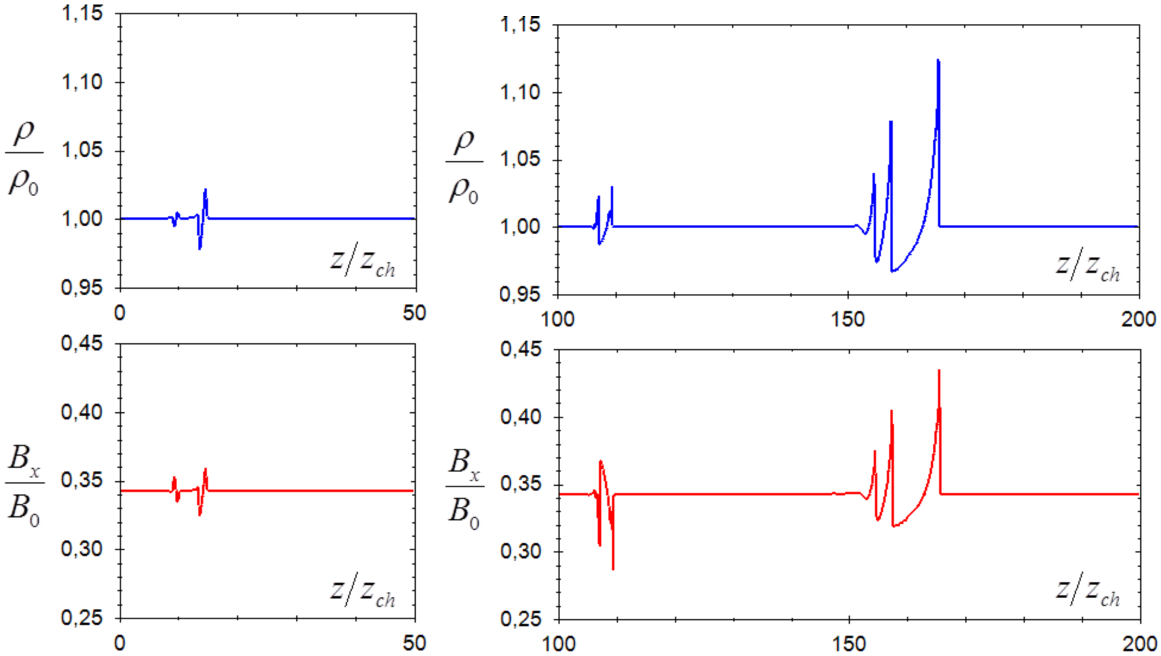
(a) (b)

Figure 3: Perturbations of density and magnetic field caused by the fast and slow magnetoacoustic waves in the moment (a) $t = 10 t_{ch}$ and (b) $t = 110 t_{ch}$ in the plasma with the dominating magnetic tension $\beta = 0.85$.

In order to show the opposite amplification properties of magnetoacoustic waves in the plasma with the dominating pressure $\beta > 2/\gamma_\infty$ we use $\beta = 1.55$ and again choose the angle between the wave vector and magnetic field $\theta = 20^\circ$. In this simulation we use the same initial perturbation of the medium stationary state. The fast and slow magnetoacoustic pulses in the moment $t = 10 t_{ch}$ are shown in Figure 4a. In Figure 4b one can see the sequence of the amplified slow pulses and amplified fast pulses with higher magnitude in the moment $t = 110 t_{ch}$. The zero point in Figures 3 and 4 correspond to the position of initial velocity perturbation of the medium stationary state.

4 Conclusions

In this paper, we show that anisotropy caused by the external magnetic field influences not only in phase and group velocity of magnetoacoustic waves but also on their increments in thermally unstable media. Using linear analysis of the MHD equations and their numerical solution we show that the amplification of fast magnetoacoustic waves is stronger than the amplification of slow magnetoacoustic waves in plasma with the dominating pressure $\beta > 2/\gamma_\infty$ $(c_\infty > c_a)$. These features take place at all angles between the wave propagation direction and magnetic field. In plasma with the dominating magnetic tension $\beta < 2/\gamma_\infty$ $(c_\infty < c_a)$ the situation changes to opposite.



(a) (b)

Figure 4: Perturbations of density and magnetic field caused by the fast and slow magnetoacoustic waves in the moment (a) $t = 10t_{ch}$ and (b) $t = 110t_{ch}$ in the plasma with the dominating pressure $\beta = 1.55$.

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