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Redesigning Helmholtz resonators to achieve attenuation at multiple frequencies

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Abstract

Helmholtz resonators can be used to reduce narrowband noise levels at low frequencies. These resonators are generally designed to target only one frequency. In order to attenuate multiple frequencies, it is possible to use several Helmholtz resonators tuned at different frequencies and located one after the other, or to connect several Helmholtz resonators in series. However, this may have implications on the package size. In this work, the exploitation of cavity modes is investigated as an alternative approach to targeting multiple frequencies from a single Helmholtz resonator. Numerical methods are used to illustrate the influence of neck location on mode selection and resonance frequency. The influence of multiple neck openings into a single cavity is also investigated. The numerical simulations are shown to be in good agreement with experiments for various designs of multi-resonant Helmholtz resonators. Finally, by way of example, a multi-resonant system is designed where the two principle resonant frequencies are tuned independently by adjusting the location of the neck and shape of the cavity. The paper highlights a design methodology to take advantage of the cavity modes within a Helmholtz resonator in order to control multiple frequencies from a single package.

Keywords: Helmholtz silencers, Cavity resonance, Narrowband attenuation

Redesigning Helmholtz resonators to achieve attenuation at multiple frequencies

1 Introduction

Helmholtz resonators can provide narrowband acoustic attenuation. They generally consist of a volume connected to a pipe via a neck. At low frequencies these silencers can be modelled using a lumped element approach. This technique generally gives a good approximation of the attenuation frequency. To get a better estimation several authors have introduced a length correction factor (see [1-2]). This correction factor compensates for the effect of 3D evanescent waves present at the interface between the neck and the cavity. The resulting impedance of the Helmholtz resonator Z_h is (from [3])

$$Z_h = i\left\{\omega\frac{l_{eq}}{S_n} - \frac{\rho^2 c^2}{\omega V_0}\right\} + \frac{\omega^2}{\pi\rho c}, \tag{1}$$

where c is the speed of sound, ρ is the density of air and ω is the angular frequency. l_{eq} is the effective length of the neck, S_n its cross sectional area and V_0 is the volume of the cavity. The effective length is the physical length of the neck to which is added the end correction ($l_{eq} = l + 2 \times 0.85r$, [3]).

In order to target multiple frequencies, it is necessary to use several Helmholtz either in parallel (one after the other, see [4]) or in series (see [5]). However, in this paper we introduce a method to achieve multiple frequencies of attenuation from a single resonator by taking advantage of higher order modes within the cavity. Particular modes can be selected by judicious placement of the neck relative to the cavity. In the first section we derive an expression of impedance for a Helmholtz resonator with a long rectangular cavity by considering the propagation inside the cavity. This requires the axial or radial propagation in the cavity to be considered. This approach was used in [6] to understand the effect of the end correction when one dimension of the cavity is larger than the wavelength. The analytical expression is compared with experiments and numerical simulations. The latter allows the higher order modes inside the cavity to be directly visualised.

A second approach to mode selection involves the use of multiple necks connected to a single resonator cavity. The influence of neck location and cavity shape on frequency of attenuation is studied for different cases.

2 Asymmetrical Helmholtz silencer

2.1 Impedance formulation

This section aims to show that by changing the location of the neck relative to the cavity, it is possible to get an additional frequency of attenuation. This new attenuation cannot be predicted by using the impedance of the Helmholtz introduced by Equation (1) as it considers the cavity as one volume. A new expression of the Helmholtz impedance is developed here by considering the propagation inside the cavity. The geometry under consideration is shown in Figure 1.

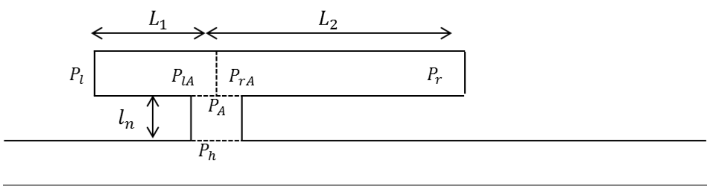

Figure 1: Schematic of Helmholtz resonator configuration. The neck has a length l_n and cross-section S_n. The cavity has a length $L_1 + L_2$ and cross-section S_{cav}.

The transfer matrix method can be used to calculate pressure and volume velocity inside the system. The pressure P_h and volume velocity U_h at the inlet of the resonator are given by:

$$\begin{pmatrix} P_h \\ U_h \end{pmatrix} = \begin{pmatrix} \cos kl_n & iZ_n \sin kl_n \\ \frac{i}{Z_n} \sin kl_n & \cos kl_n \end{pmatrix} \begin{pmatrix} P_A \\ U_A \end{pmatrix}, \tag{2}$$

where P_A and U_A represent the pressure and volume velocity at the interface between the neck and the cavity, Z_n is the characteristic acoustic impedance of the neck ($Z_n = \rho c / S_n$) and l_n is the length of the neck.

At the interface between the neck and the cavity, the same system of equations describes both sides of the cavity which are each considered as a pipe with closed end. The acoustic volume velocity at each end equals zero ($U_{l/r} = 0$). On left-hand part of the cavity:

$$\begin{pmatrix} P_{lA} \\ U_{lA} \end{pmatrix} = \begin{pmatrix} \cos kL_1 & iZ_{cav} \sin kL_1 \\ \frac{i}{Z_{cav}} \sin kL_1 & \cos kL_1 \end{pmatrix} \begin{pmatrix} P_l \\ 0 \end{pmatrix}, \tag{3}$$

where Z_{cav} is the characteristic impedance of the cavity ($Z_n = \rho c / S_{cav}$), L_1 is the distance from the neck to the closed end on the left and P_l is the pressure at this end. In the same way, from the right hand side, we get:

$$\begin{pmatrix} P_{rA} \\ U_{rA} \end{pmatrix} = \begin{pmatrix} \cos kL_2 & iZ_{cav} \sin kL_2 \\ \frac{i}{Z_{cav}} \sin kL_2 & \cos kL_2 \end{pmatrix} \begin{pmatrix} P_r \\ 0 \end{pmatrix}. \tag{4}$$

Continuity of pressure at the neck-cavity intersection gives $P_A = P_{lA} = P_{rA}$, and conservation of volume velocity leads $U_A = U_{lA} + U_{rA}$.
Finally, rearranging these equations, it gives an expression for the impedance of the Helmholtz silencer $Z_h = P_h/U_h$:

$$Z_h = iZ_n \frac{Z_n \sin\big(k(L_1 + L_2)\big) \sin(kl_n) - Z_{cav} \cos(kl_n) \cos(kL_1) \cos(kL_2)}{Z_n \sin\big(k(L_1 + L_2)\big) \cos(kl_n) + Z_{cav} \sin(kl_n) \cos(kL_1) \cos(kL_2)}. \tag{5}$$

Using the transfer matrix method, an expression for the transmission loss can be obtained using the following expression [3] :

$$TL_{dB} = 20 \log_{10} \left(\frac{1}{2} \left(2 + \frac{Z_{pipe}}{Z_h} \right) \right), \tag{6}$$

where Z_{pipe} represents the characteristic impedance of the main pipe. Figure 2 shows the transmission loss for the case where there is no offset of the cavity ($L_1 = L_2$), shown in green. The red line ($L_1 < L_2$) illustrates the influence of an offset and introduces an additional frequency of attenuation around 3500 Hz.

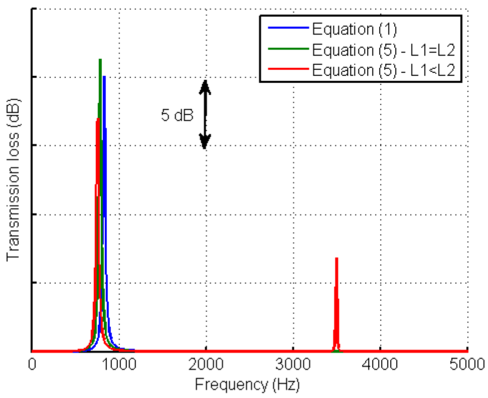

Figure 2: Transmission loss of a Helmholtz silencer for two different positions of the neck relative to the cavity.

It can also be observed that using this model, the first frequency of attenuation is modified as the offset increases. This effect was used to define a modified length correction of the neck in Equation (1) by Selamet [7] and Chanaud [8].

2.2 Experimental observation

In order to observe the influence of asymmetry on Helmholtz silencers, transmission loss experiments were conducted. A 30 mm wide pipe with square cross-section was used for the experimental configuration. The Helmholtz resonator was composed of a cylindrical neck of 2mm length and 12mm diameter. The cavity was a square duct of 14mm width and 86 mm length (see Figure 3).

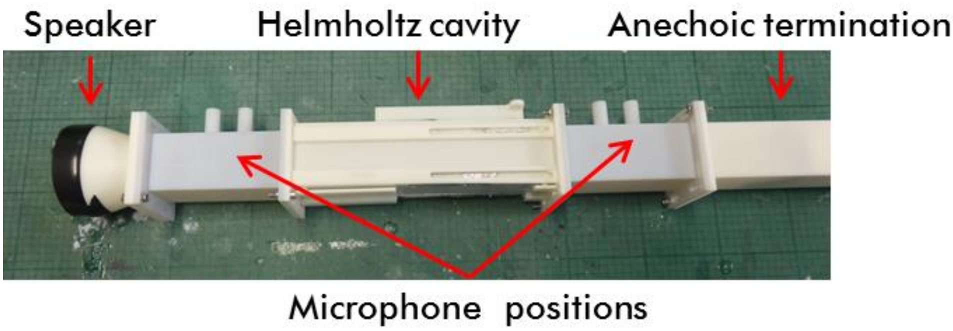

Figure 3: Picture of the test rig.

The position of the neck within the cavity was varied with an offset 0, 20 and 37mm from the centre of the cavity. The pressure was measured at two points before the silencers and two points after using microphones. This allowed calculation of the transmission loss using wave decomposition.

The measured transmission losses for the varying resonator neck positions are shown in Figure 4.

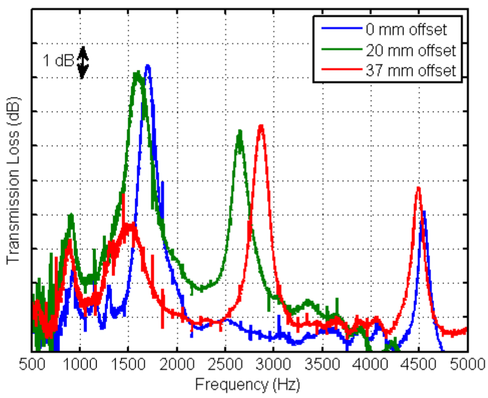

Figure 4: Experimental transmission loss for different offset. 0mm offset corresponds to the symmetrical case.

It shows that for all configurations, the Helmholtz resonator gives attenuation around 1500Hz. However, the level of attenuation is reduced for the largest offset, this is thought to be due to damping inside the cavity. When asymmetry is introduced (offset different from 0), a new frequency of attenuation appears around 2800Hz. A third frequency of attenuation can be observed around 4500Hz, but not for the case where the offset is 20mm.

2.3 Analytical and numerical simulation

The transmission loss can be computed using Equation (6). Considering the cross section in the main pipe, the plane wave cut off frequency is around 6kHz. Figure 5 shows the transmission loss obtained using Equation (6). It presents the same behaviour as observed experimentally, however the frequencies are different. This is thought to be due to the choice of end corrections.

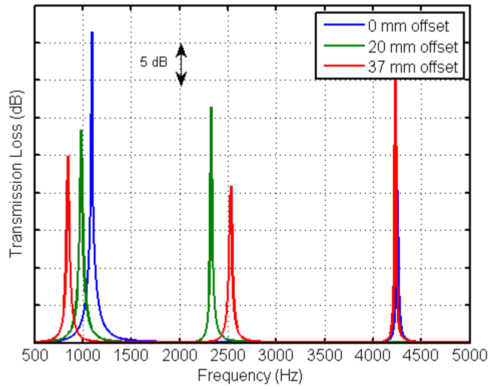

Figure 5: Transmission loss for the three configurations using Equation (6).

This model provides useful insight but is limited. In order to get a better model and to understand what happens when asymmetry is introduced in the Helmholtz resonator, numerical simulations were conducted using an FEA package. The same geometry was used and the damping was taken into account using a model for a square duct of small cross section. Plane wave conditions were defined at both ends of the main pipe with a plane pressure wave at the inlet side. The mesh was built so that there were at least 8 elements per wavelength.
The transmission loss was computed, as illustrated in Figure 6, using the ratio of acoustic energy at the inlet and outlet of the pipe. Comparing with experimental results from Figure 4, peaks of attenuation around 2800 and 4500 Hz align with the experiments in the correct geometries. However the first attenuation frequency is still underestimated. It can be seen that there are discrepancies between the levels of attenuation in the experiment and numerical simulation. It is thought that this could be due to damping effects within the pipe which are difficult to model and could be affected by the surface of the rig in the experiment.

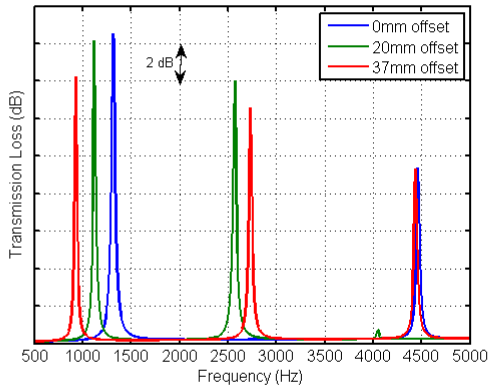

Figure 6: Transmission loss introduced by the Helmholtz resonator for different offset of the neck calculated with FEA. 0mm offset corresponds to the symmetrical case.

Additionally, the first frequency of attenuation seems to be shifted towards lower frequencies for increasing offset of the neck as predicted by the impedance model and observed in [7]. This effect was also observed experimentally but was not as severe. In order to understand what happens at the second frequency of attenuation, Figure 7 shows the computed pressure field at this frequency for the case where the offset is 20mm. It can be seen in Figure 7 that at 2570 Hz the pressure field in the cavity corresponds to an excitation of its second mode. In the case, where the neck is centred relative to the cavity, this mode cannot be excited as the neck is located at a node of pressure for that mode.

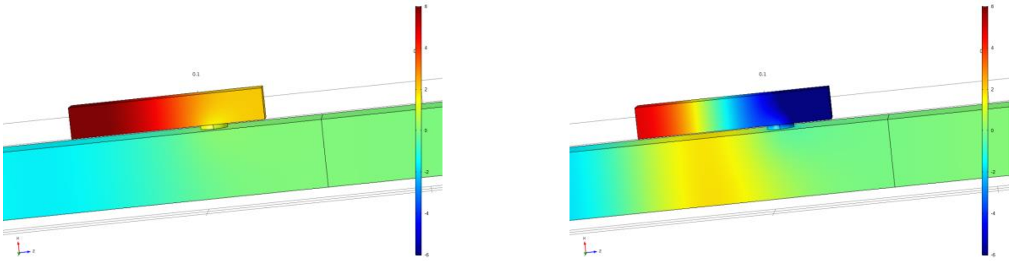

Figure 7: Pressure field in the system for an offset of 20mm. Left - first attenuation frequency (1120Hz). Right – second attenuation frequency (2570Hz).

Similarly for the attenuation at 4500 Hz a higher mode of the cavity is excited. This attenuation does not appear for an offset of 20 mm for which the neck is located at a node of pressure for that mode.

3 Helmholtz resonator with several necks

The previous section shows that the position of the cavity relative to the neck allows a modification of the first resonance frequency and selection of an additional resonance frequency.

By using several necks, we show it is possible to modify these frequencies further. In this section the Helmholtz resonator consists of several necks connected to the same cavity. The cavity encloses the main pipe as shown in Figure 8.

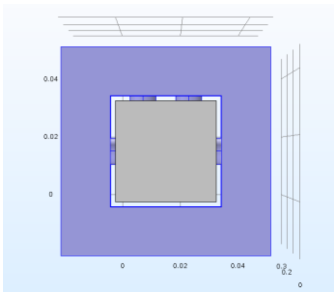

Figure 8: New configuration with the cavity and necks (blue) around the main pipe (grey).

3.1 Influence of neck location

In order to study the influence of the neck positions, different configurations were studied. Either 1, 2 or no connections to the main pipe to the cavity are placed on each of the four sides as described in Table 1.

Table 1: Configurations with different neck positions around the square pipe. It gives the number of holes on each wall of the main pipe.

Configuration	Left wall	Top wall	Right wall	Bottom wall
1111	1	1	1	1
1210 (Figure 8)	1	2	1	0
2200	2	2	0	0
2020	2	0	2	0

The first configuration '1111' corresponds to the symmetrical case. The insertion loss was calculated both experimentally and numerically and results are shown in Figure 9.

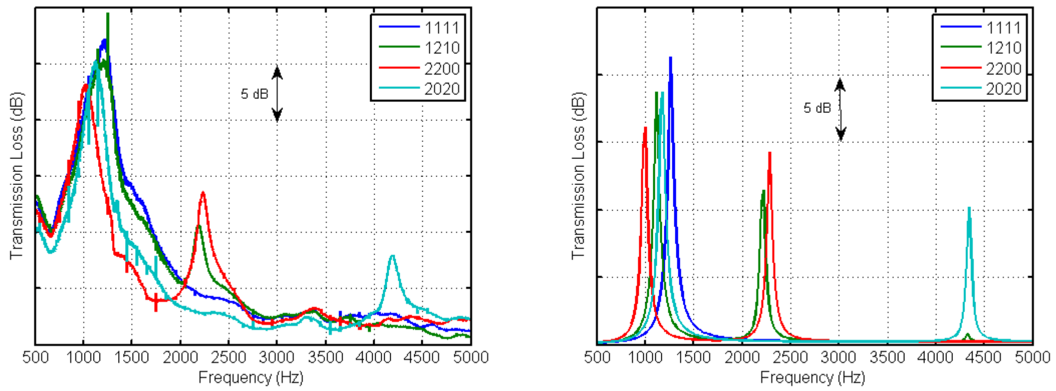

Figure 9: Left - Experimental transmission loss for different neck positions. Right - Numerical transmission loss.

Figure 9 shows that for all four configurations attenuation is obtained around 1200Hz. This frequency seems to be modified for each configuration. However it is difficult to see any trend in the experimental data, while numerical simulation indicates that this frequency decreases as the asymmetry increases. This is still in agreement with what was discussed in the previous section. Additionally, configurations '1210' and '2200' present attenuation around 2200Hz, while the configuration '2020' present an attenuation frequency around 4300Hz. Figure 10 shows the mode excited at both these frequencies.

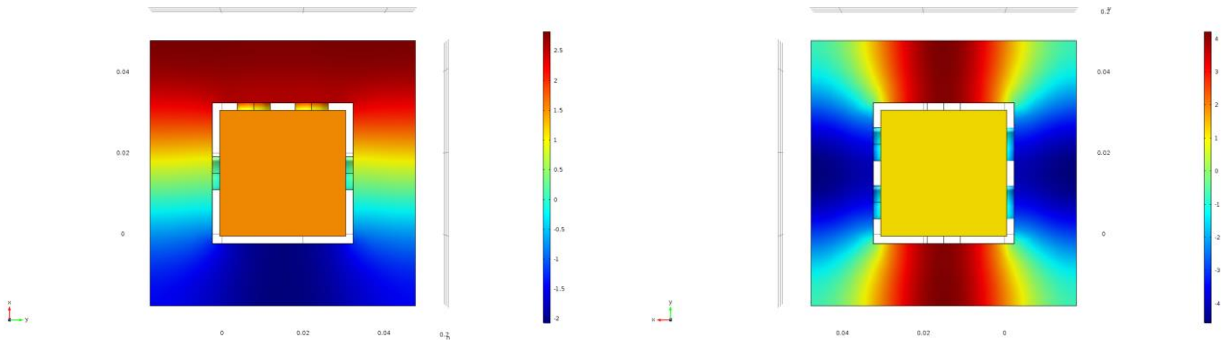

Figure 10: Left – Cavity pressure field for second attenuation frequency ('1210' configuration at 2220Hz). Right – Cavity pressure field for third attenuation frequency ('2020' configuration at 4340Hz).

3.2 Influence of cavity shape

A second parameter of interest is the effect of the shape of the cavity on the frequencies of attenuation. To investigate this, four different cavities of the same volume, but different aspect ratio were built and tested. Their dimensions are given Table 2 and pictures are shown in Figure 11.

Table 2: Dimensions of the four cavities.

Cavity	Depth (mm)	Length (mm)
1	20	12
2	15	20
3	10	34
4	5	70

Figure 11: Pictures of studied cavities. From left to right cavity 1 to 4.

They are tested using an asymmetrical distribution of necks ('1210') in order to study the effect on the second frequency of attenuation. The resulting transmission losses are plotted in Figure 12.

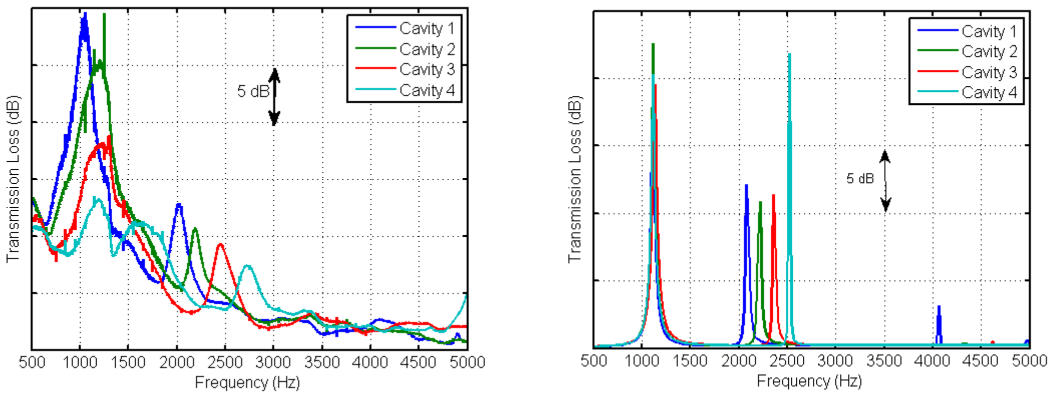

Figure 12: Left - Experimental transmission loss for different position of the necks. Right - Numerical transmission loss.

It can be seen that the first frequency of attenuation remains approximately constant; however, transmission loss decreases with increasing cavity length. The second frequency of attenuation is modified as the cavity shape changes. This set of experiments shows that it could be possible to set a first frequency of attenuation by choosing the right volume and neck shape and then to tune the second one (to a certain degree) by modifying the shape of the cavity. In this case, experimental results would suggest that there is also a compromise to be found as for extreme aspect ratio, the attenuation at the first frequency is reduced significantly.

4 Conclusions

In this paper, it was demonstrated how higher order modes can be selected to target multiple frequencies of attenuation from a single Helmholtz resonator. This can be useful for applications where multiple tones are to be attenuated and space is limited.

It was observed that the first resonance frequency is sensitive to the asymmetry of the resonator, while the higher order resonance is also sensitive to the cavity shape. By using these techniques the designer will be allowed significant freedom to tune their resonator to a variety of target frequencies.

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