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Inverse acoustic characterization of rigid frame porous materials from impedance tube measurements

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Abstract

We will present a method for the inverse characterization of rigid frame porous materials using audible frequency acoustic measurements in an impedance tube 3 cm in diameter. We recover the six acoustical parameters of the Johnson-Lafarge model, namely porosity, tortuosity, viscous and thermal characteristic lengths and flow and thermal resistivities. The proposed method is based on a minimization process, where the quantities of interest are found as the minimizing values for the difference between measured and modeled density and compressibility. A scattering matrix formulation is used to obtain the reflection and transmission coefficients R and T, found as the elements of the scattering matrix, and to obtain the effective density and compressibility in the range of 200 - 6500 Hz. Five different porous materials with flow resistivity ranging from 2,000 to 60,000 Ns/m⁴ are tested and the results of the inversion process are compared to direct measurements of the acoustical quantities as well as to an already established recovery method developed by Olny and Panneton. The results are also validated against measurements with a rigid backing. It is found that the proposed method can be used to recover the material parameters quickly and reliably.

Keywords: porous materials, impedance tube measurements, inversion

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1 Introduction

This paper presents a method for characterizing sound absorbing porous media. The characteristics of a medium can be thought of as the parameters and their values that are present in a model describing the material. Here the focus is on models that consider the material frame as rigid and motionless, when medium can be modeled as an equivalent fluid. Good introductions to different models describing an equivalent fluid can be found for example in Refs. [1], [2] and [3].

The method utilized in this paper uses a Johnson[4]-Champoux-Allard[5] semi-phenomenological model and is based on recovering the parameters from the material's complex density $\tilde{\rho}$ and compressibility \tilde{K} . With the proposed method it is possible to recover porosity ϕ , tortuosity α_{∞} , characteristic viscous and thermal lengths Λ and Λ' , flow resistivity σ and thermal resistivity σ' . Inversion for the parameter recovery is based on fitting analytical curves, that are calculated from the estimated parameters, to measured curves in the least-squares sense. Equations for flow and thermal resistivities σ and σ' are rewritten in a more physical way to increase sensitivity of the inversion.

2 THEORY

Consider an isotropic, homogeneous, open-cell porous medium whose skeleton is assumed to be motionless so that it can be modeled as an equivalent fluid. Such a medium can be characterized by an equivalent dynamic density $\tilde{\rho}_{eq}$ and a dynamic bulk modulus \tilde{K}_{eq} . This assumption is valid when the fluid-structure coupling is weak and the wavelength is much larger than the characteristic dimensions of the pores [1].

In the rigid frame model sound waves are attenuated due to viscous and thermal losses which are represented by $\tilde{\rho}_{eq}$ and \tilde{K}_{eq} , respectively. The tilde over certain functions indicates that they are complex and frequency-dependent. Several semi-phenomenological models have been developed for relating the acoustical behavior of the material to its geometrical properties. These include the Johnson *et al.*[4], Champoux-Allard [5], Pride *et al.*[6] and Lafarge [7] models.

The equivalent density of the porous medium can be written as

$$\tilde{\rho}_{eq} = \frac{\rho_0}{\phi} \tilde{\alpha}(\omega), \quad (1)$$

where ρ_0 is the density of the saturating fluid, ϕ the open porosity and $\tilde{\alpha}(\omega)$ the dynamic tortuosity. In a same fashion the equivalent bulk modulus can be written like

$$\tilde{K}_{\text{eq}} = \frac{\gamma P_0}{\phi} \left(\gamma - \frac{\gamma - 1}{\tilde{\alpha}'(\omega)} \right)^{-1}, \quad (2)$$

where P_0 is the static pressure and γ the specific heat ratio. The parameter $\tilde{\alpha}'(\omega)$ has been defined as a homologue to $\tilde{\alpha}(\omega)$, representing the thermal tortuosity [7].

2.1 Johnson *et al.* model

The Johnson *et al.* model [4] is a semi-phenomenological model used to describe the complex density of the porous medium. In this model, adopting the $e^{-i\omega t}$ convention, the dynamic tortuosity is written as

$$\tilde{\alpha}(\omega) = \alpha_\infty \left[1 + \frac{i\phi\sigma}{\rho_0\alpha_\infty\omega} F(\omega) \right], \quad (3)$$

where $F(\omega)$ is a shape function defined by

$$F(\omega) = \sqrt{1 - i\omega\eta\rho_0 \left(\frac{2\alpha_\infty}{\phi\sigma\Lambda} \right)^2}, \quad (4)$$

where η is the viscosity of the saturating fluid, α_∞ , σ and Λ are the tortuosity, flow resistivity and viscous characteristic length[8] of the porous medium, respectively.

2.2 Champoux-Allard model

The Champoux-Allard model [5] is a semi-phenomenological model that can be used to describe the dynamic bulk modulus of the porous medium. In this model the thermal tortuosity is written as

$$\tilde{\alpha}'(\omega) = 1 + \frac{i\phi\sigma'}{\rho_0\alpha_\infty\text{Pr}\omega} G(\omega), \quad (5)$$

where $G(\omega)$ is given by

$$G(\omega) = \sqrt{1 - i\omega\eta\rho_0\text{Pr} \left(\frac{2\alpha_\infty}{\phi\sigma'\Lambda'} \right)^2}, \quad (6)$$

where Λ' is the thermal characteristic length, σ' is the thermal resistivity and Pr is the Prandtl number.

3 RECOVERY METHOD

Using the Johnson and Champoux-Allard models to describe the dynamic functions $\tilde{\rho}_{\text{eq}}$ and \tilde{K}_{eq} of the equivalent fluid, there are six acoustical parameters to estimate, namely $\phi, \alpha_\infty, \Lambda, \Lambda', \sigma$ and σ' . However, by making a substitution of [4, 9]

$$c = \frac{\Lambda^2 \sigma \phi}{8\eta \alpha_\infty}, \quad c' = \frac{(\Lambda')^2 \sigma' \phi}{8\eta \alpha_\infty}, \quad (7)$$

new expressions can be written for $\tilde{\rho}_{\text{eq}}$ and \tilde{K}_{eq} that are dependent on less parameters, namely $\tilde{\rho}_{\text{eq}} = \tilde{\rho}_{\text{eq}}(\phi, \alpha_\infty, \Lambda^2, c)$ and $\tilde{K}_{\text{eq}} = \tilde{K}_{\text{eq}}(\phi, (\Lambda')^2, c')$. This is physically more valid since in reality the compressibility does not depend on tortuosity. Moreover, now there is only one parameter that $\tilde{\rho}_{\text{eq}}$ and \tilde{K}_{eq} both depend on: the porosity ϕ . Recovery of the thermal resistance σ' is possible because the considered frequency range includes frequencies between the isothermal and adiabatic regime, where the effect of σ' is the largest. The modified dynamic tortuosities read

$$\tilde{\alpha}(\omega) = \alpha_\infty \left[1 + i \frac{8\eta c}{\omega \rho_0 \Lambda^2} \sqrt{1 - i \rho_0 \omega \frac{\Lambda^2}{16\eta c^2}} \right], \quad (8a)$$

$$\tilde{\alpha}'(\omega) = 1 + i \frac{8\eta c'}{\omega \rho_0 (\Lambda')^2 \text{Pr}} \sqrt{1 - i \rho_0 \omega \text{Pr} \frac{(\Lambda')^2}{16\eta (c')^2}}. \quad (8b)$$

Parameter inversion is performed by finding the minimum of the L_2 norm of the difference between measurements of $\tilde{\rho}_{\text{eq}}$ and \tilde{K}_{eq} and their analytically constructed counterparts as.

$$f(\omega; \theta) = \left\| \tilde{\rho}_{\text{eq}}^{\text{meas}}(\omega) - \tilde{\rho}_{\text{eq}}^{\text{model}}(\omega; \phi, \alpha_\infty, \Lambda^2, c^2) \right\|_2 + \left\| \tilde{K}_{\text{eq}}^{\text{meas}}(\omega) - \tilde{K}_{\text{eq}}^{\text{model}}(\omega; \phi, (\Lambda')^2, (c')^2) \right\|_2, \quad (9)$$

where θ denotes all the acoustic variables and ω is a vector containing a discrete set of relevant range of frequencies. The cost function $f(\omega; \theta)$ must be formed as a sum of the two functions $\tilde{\rho}_{\text{eq}}$ and \tilde{K}_{eq} because they both contain information of ϕ . That way, when all the other parameters are independent, a good estimate for ϕ will be acquired. One way to solve the minimization problem

$$\hat{\theta} = \arg \min_{\theta} f(\omega; \theta), \quad (10)$$

is to use the Nelder-Mead simplex algorithm [10], which is an iterative method that does not need numerical or analytical gradients. During inversion the parameters are bounded to reasonable values, such as $\phi \in [0, 1]$ and $\alpha_\infty \in [1, 3]$.

4 RESULTS

The presented method was tested with five different porous samples, each with a diameter of 30 mm and thickness varying between 16 mm to 40 mm. The materials were soft polyurethane foam, melamine foam, felt, normal glass wool and Isover[®] Calibel glass wool, see Figure 2. The proposed method is compared to measurements of the acoustical properties (flow meter [11], ultrasonic measurements[12]) as well as to recovery methods proposed in Refs. [13] and [14]. With the ultrasonic measurement Λ' or σ' were not recovered.

Figure 1 presents all the measured curves of R , T , $\tilde{\rho}_{eq}$ and \tilde{K}_{eq} along with the reconstructed curves based on the parameter values obtained from the inversion process. Real and imaginary parts are presented separately. Finally, the figure also shows an analytically calculated reflection coefficient for a case of porous material backed with a rigid wall, along with the corresponding measurement.

The ripples in the measured curves that are visible especially in the wools around 2000 Hz are caused by Biot waves [15]. They result from motion of the frame, which is not accounted for in the currently used model. However, the overall effect of the Biot waves on the reconstruction is not too large since the peaks tend to be symmetric around the reconstructed curves.

Table 1: Recovered values for the tested materials using the method proposed in this paper, the method proposed in Refs. [13] and [14] (Olny&Panneton), and other measurements.

Material							
L (mm)	Method	ϕ	α_∞	Λ (μm)	Λ' (μm)	σ (Nsm^{-4})	σ' (Nsm^{-4})
SPF 39.8 ± 0.2	Present	1.00	1.04	275	491	2060	1420
	O&P	-	1.07	329	448	1810	1480
	Meas	0.98	1.05	289	-	1800	-
Melamine 31.3 ± 0.1	Present	1.00	1.00	137	178	7570	4550
	O&P	-	0.79	67	166	7480	4070
	Meas	0.97	1.00	166	-	8600	-
Felt 16.5 ± 0.5	Present	1.00	1.00	36	110	25500	9570
	O&P	-	0.77	33	118	21500	8770
	Meas	1.00	1.00	45	-	28000	-
Wool 22.6 ± 0.3	Present	0.93	1.00	59	140	32400	20200
	O&P	-	1.16	53	190	27000	11900
	Meas	1.00	1.00	64	-	24000	-
Calibel 28.3 ± 0.3	Present	0.97	1.00	36	120	47800	29400
	O&P	-	1.38	26	91	38500	16900
	Meas	0.97	1.01	37	-	57000	-

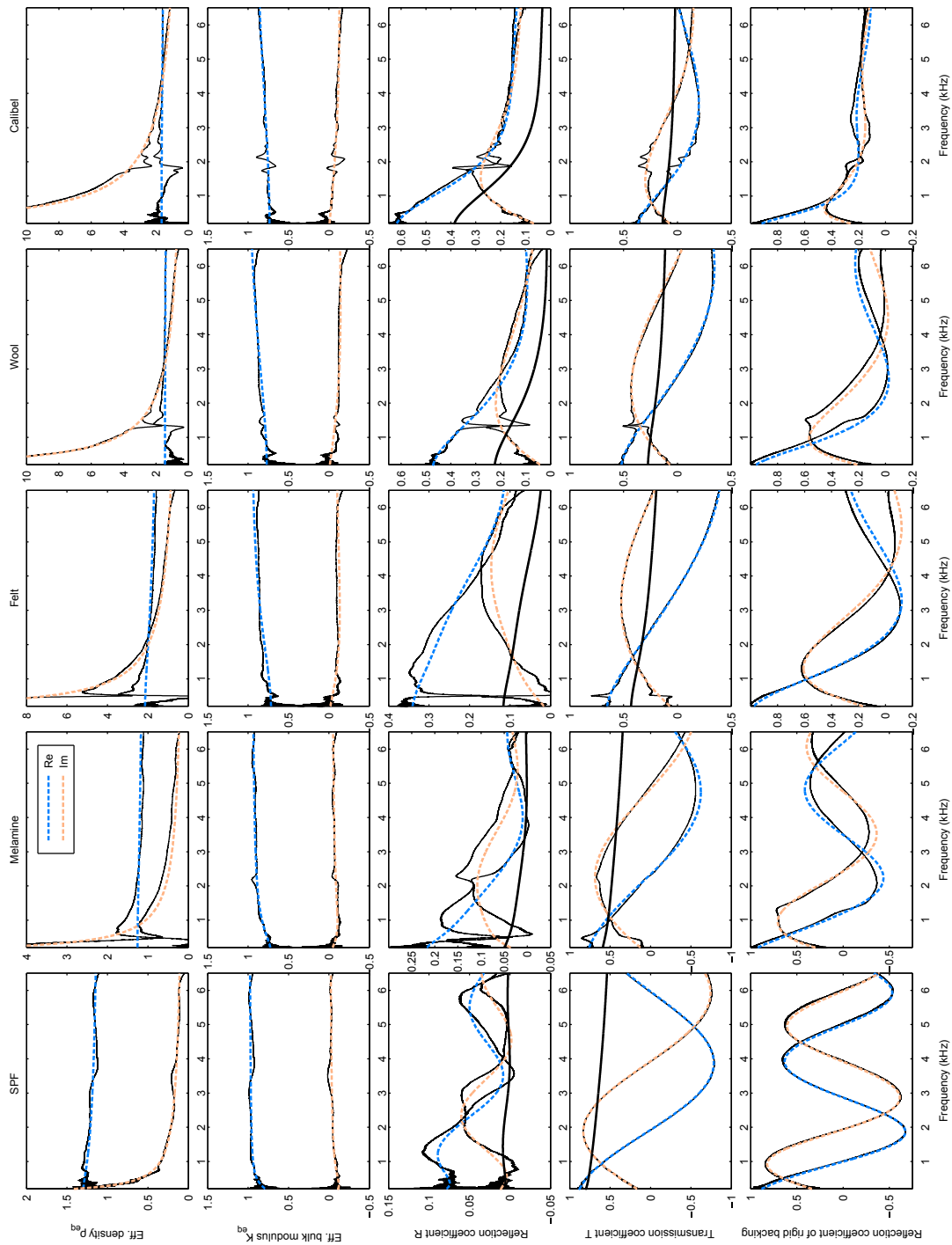


Figure 1: Real and imaginary parts for measured and recovered values for ρ , K , R and T . Thin black lines are the measurements, dotted lines are from the model with the recovered parameters. The bottom row shows a reconstruction and measurement against a rigid backing. The thicker black lines in R and T measurements are $|R|^2$ and $|T|^2$, respectively. Notice that the y -axis changes from figure to figure for better visibility.

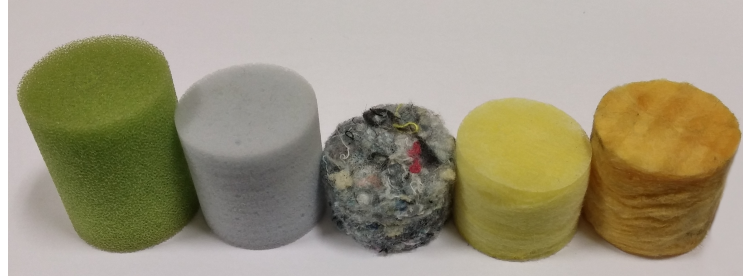


Figure 2: Samples used in this work. From the left: 1. Soft polyurethane foam (SPF), 2. Melamine foam, 3. Felt, 4. Glass wool, 5. Isover Calibel wool.

5 Conclusions

In this work, a minimization method for recovering the acoustical parameters in the Johnson-Champoux-Allard model, using the measured equivalent dynamic density $\tilde{\rho}_{eq}$ and dynamic bulk modulus \tilde{K}_{eq} , was presented. The recovery algorithm minimizes the L_2 norm of the difference between the measured and analytically calculated curves. This method allowed the simultaneous recovery of all six parameters of interest. For the inversion process, a change of variables was made to represent the density and compressibility functions in a physically more correct way, that reduced the interdependence and increased the inversion sensitivity of the parameters.

The obtained values were compared to direct and ultrasonic measurements and to a recovery process proposed in Refs. [13] and [14]. Results indicated that the presented method can be used to recover all six parameters in the model with good accuracy. The method can also be applied quickly since the recovery process is autonomous, and the parameters are recovered simultaneously. The presented method recovers Λ and Λ' independently, and allows also the recovery of the thermal resistivity σ' .

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