Abstract

This paper is concerned with the transient response prediction of an impulsively excited structure. The frequency range typical for such problem is well beyond the practical limits of the standard finite element methods. In practice, the transient statistical energy analysis is often applied to the transient problem, but transient statistical energy analysis models and solvers are not readily available in some cases. Solving the problem in the framework of finite element analysis is still of practical value. The present work extends earlier scaling approaches for the steady state analysis to the transient response prediction. Based on the transient statistical energy analysis, a general scaling law is derived from the similitude of the governing equations of the scaled model and the original system. Because the scaled model is of reduced complexity and has similar dynamics to the original system, it can be efficiently solved with a transient finite element solver to simulate the impulsive response of the original system. Numerical examples involving coupled plate structures validate the applicability of the proposed approach. Due to its computational efficiency, the approach offers a way to extend the finite element methods to the impulsive response analysis.

Keywords: TSEA; transient response; scaling approach; similitude
Transient response prediction of an impulsively excited structure using a scaling approach

1 Introduction

Many engineering structures may be subject to excitations with large amplitudes over a short time duration. These impulsive excitations can cause structural failure or unwanted noise especially in the high-frequency band. To better describe their dynamics, transient responses from an energetic point of view are often used. Although many methods can predict high-frequency responses in the steady state, only a few deal with the prediction of the high-frequency responses in the transient state. An earlier work on the subject was published by Manning and Lee [1] in the context of transient statistical energy analysis (TSEA). They proposed a method based on a steady state power balance equation to deal with the mechanical shock transmission. Reasonable agreement between measured and predicted vibration responses was shown for a beam-plate system with a transient structure-borne excitation. However, the application of steady state coupling loss factors (CLFs) in the transient condition requires further justification. Lai and Soom [2] proposed using time-varying CLFs to describe the modal interactions in the case of impulsive excitation. More recently, Pinnington and Lednik [3, 4] investigated the transient responses of a two-oscillator system and a coupled-beam system. It is shown that TSEA gives good estimates of peak energy transmitted for modal overlaps less than unity.

As an alternative method to TSEA, thermal analogy is used to obtain the energy distribution in subsystems. Nefske and Sung [5] generalized the thermal analogy in the time domain, but only steady state results have been shown. Ichchou et al. [6] pointed out that the analogy between the vibrational energy flow and the thermal flow is no longer valid for the transient state. They proposed a transient local energy approach (TLEA) where the governing equation is a telegraph-type equation for one-dimensional damped structures. Sui et al. [7] compared time domain results for TLEA and TSEA to the exact solution of the two-oscillator system which shows better agreement for peak level and peak location for TLEA than TSEA.

Although TSEA and TLEA have been validated on some simple structures, their extension in the time domain lacks a rigorous theoretical foundation. Recently, Savin et al. [8, 9] proposed a transport model for high-frequency vibrational power flows in coupled heterogeneous structures. It is shown that the energy densities associated to high-frequency elastic waves in each single structure satisfy transport equations when the correlation lengths of the heterogeneous materials comparable with the wavelength. The energy densities are coupled at the junctions where the reflection and transmission phenomena of the energy fluxes are described by power flow reflection and transmission operators. After long times and/or propagation distances, the transport model evolves into a diffusion model when the energy rays are fully mixed due to scattering on the heterogeneities or multiple reflections on the boundaries [10].

The above energetic approaches feature that the primary variables are the quadratic terms related the subsystem energy or energy density. Their merits lie in the reduced computational cost and automatically taking into account the unavoidable uncertainties in the high-frequency
band, however, it is very challenging to compute the transient high-frequency responses of a complex structure due to lack of a mature software to implement these approaches. On the other hand, finite element (FE) methods are well established and transient solvers are readily available in most FE softwares. Although FE methods are limited to low frequencies due to intensive computation, efforts have been made in the past decades to extend the their applications to high frequencies.

It is an interesting observation that two different systems may yield similar responses in the high-frequency band. If a scaled model is available which is dynamically similar to the original system and has a much lower modal density, one can build an FE model with much fewer degrees-of-freedom and solve it with relatively low computational cost. The responses of the scaled model can be used to estimate those of the original system in the high-frequency band. There are mainly two ways to construct the scaled model. De Rosa et al. [11, 12] proposed asymptotic scaled modal analysis (ASMA) to obtain an acceptable approximation of high-frequency vibrations using a reduced modal base. The scaled model in ASMA is determined using an energy distribution approach [13], resulting in a reduction in the dimensions not involved in energy propagation and accordingly an increase of the original damping loss factors. Li [14, 15] proposed to use statistical energy analysis (SEA) similitude to build the scaled model. In this paper, the approach is extended to deal with the transient problem. Besides the structural parameters, the time variable is also scaled to achieve the similitude in the time domain. The scaled model usually has a larger damping and a faster time variable, making the energy dissipate in subsystems and transmit to adjacent subsystems at a faster rate than in the original system. Therefore, one can obtain the transient responses from the scaled model in a shorter simulation time, which is then scaled back to the original time to obtain the final responses.

Preliminary results of the transient scaling approach are presented. In Sec. 2, the general scaling laws are derived from TSEA governing equations. Then specific forms of the scaling laws are derived in Sec. 3 for a line coupled plates. Numerical validation is presented in Sec. 4 and some conclusions are given in Sec. 5 about the application of the transient scaling approach in the high-frequency response prediction in the time domain.

2 General scaling laws

TSEA is based on the instantaneous power flow balance among coupled subsystems. Although the justification of TSEA is out of the scope of this paper, one could list some basic prerequisites for the successive application of TSEA. One group of prerequisites comes from SEA, which requires that the system can be partitioned into a set of weakly coupled subsystems and each subsystem carries a diffusive field. The latter requirement is likely to be satisfied when the subsystems are lightly damped and have many modes. The other group of prerequisites comes from the fact that TSEA describes a diffusive type of high-frequency vibrations, which requires multiple scatterings on the heterogeneities and reflections on the boundaries to justify a diffusive vibrational energy field. This requirement is usually met after the very beginning of vibrations. Hereafter the addressed problem is assumed to be in the diffusion regime and all
the above prerequisites are met. Furthermore, we assume that steady state CLFs can be used in TSEA with acceptable accuracy.

The TSEA governing equation for the \( j \)th subsystem at a band center frequency \( \omega \) is

\[
d\frac{E_j}{dt} + \omega \eta_j E_j + \sum_k \omega \eta_{jk} n_j (E_j/n_j - E_k/n_k) = P_{in,j},
\]

where \( \eta_j \) is the loss factor of subsystem \( j \), \( E_j \) is the instantaneous subsystem energy, \( n_j \) is the modal density of the subsystem, \( P_{in,j} \) is the power input to the subsystem from external excitations, and \( \eta_{jk} \) is the CLF from subsystem \( j \) to subsystem \( k \). In practice, the TSEA equations can be solved by a time difference method. At each time step, the discrete form of Eq. (1) is essentially a steady state SEA power balance equation and time-averaged subsystem energy is solved for that time interval. The discretization time interval should be long enough to allow the time-averaged net power flow to occur and short enough to make the stationary representation of the subsystem energy accurate [16].

Equation (1) can be rewritten in a matrix form as

\[
N \frac{de}{dt} + Se = P,
\]

where \( N \) is a diagonal matrix with the \( j \)th diagonal element as \( n_j \), \( S \) is a matrix with the \((j,k)\) element as \((\mu_j + \sum l \mu_{jl}) \delta_{jk} - \mu_{kj} \), in which \( \mu_j = \omega \eta_j n_j \), \( \mu_{jk} = \mu_{k j} = \omega \eta_{jk} n_j \), and \( \delta_{jk} \) is the Kronecker delta, \( e \) is a vector with the \( j \)th component as the modal energy \( E_j/n_j \), and \( P \) is a vector with the \( j \)th component as \( P_{in,j} \).

Similarly, the TSEA governing equations for the scaled model are

\[
\hat{N} \frac{d\hat{e}}{d\hat{t}} + \hat{S} \hat{e} = \hat{P},
\]

where the caret stands for the scaled model. Denote \( \gamma \) as the scaling factor and its subscript refers to the corresponding scaled parameter. For example, the power input vector is scaled as

\[
\hat{P}(\hat{t}) = \hat{P}(\gamma t) = \gamma P(t).
\]

It can be derived from Eqs. (2) and (3) that the TSEA governing equations are scaling invariant when all subsystems follow the scaling laws given by

\[
\gamma_\eta \gamma = 1,
\]

and

\[
\gamma_\mu = \gamma_\eta \gamma = \gamma_P,
\]

where no scaling in the frequency parameter \( \omega \) is assumed. The subscript denoting a term associated with a subsystem is dropped in Eqs. (4) to (6) because the same scaling laws are applied to all subsystems.

The scaling law (5) is unique for the transient analysis. It implies that the time variable should be scaled inverse-proportionally to the (coupling) loss factor. Since the scaled model usually has a larger damping than that of the original system, the subsystem energy will evolve faster
The scaling law (6) implies that the loss factor $\eta_j$ and the coupling loss factor $\eta_{jk}$ scale by the same factor. For a complex system consisting of $N$ coupled statistical subsystems, it means each loss factor $\eta_j$ needs to scale by the same factor as $(N-1)$ CLFs $\eta_{jk}, (k \neq j)$. This requirement distinguishes the present scaling approach from those derived for an isolated system. At the first sight, it seems impossible to fulfill the requirement for a complex coupled structure. The key to resolve the problem is to isolate each SEA subsystem from the rest of the system with an adapter structure. A procedure to implement the scaling law (6) was developed in [17].

The auxiliary requirements of the general scaling laws derived for the steady state analysis also apply to the transient case. The scaling factor $\gamma_n$ has a lower limit to guarantee that each subsystem contain a number of resonant modes over the analysis band of interest and the modal overlap factor is large enough. One should choose an appropriate value for $\gamma_n$ to balance the computational efficiency and the accuracy of the scaled model.

## 3 Specific scaling forms for line coupled plates

In this section, specific scaling forms are derived for the case of line-coupled plates. Each plate is modeled as a SEA subsystem and the coupling loss factor between SEA plates $j$ and $k$ through a line junction is [18]

$$\eta_{jk} = \frac{L_{jk} c_{g,j}}{\pi \omega A_j} \tau_{jk},$$

where $L_{jk}$ is the length of the line junction, $c_{g,j}$ is the group velocity of plate $j$, $A_j$ is the area of the plate, and $\tau_{jk}$ is the transmission coefficient, which can be approximated by [19]

$$\tau_{jk} = \tau_{jk}(0) \frac{2.754 X}{1 + 3.24 X},$$

where $X = h_j/h_k$ is the ratio between the thicknesses of the plates and

$$\tau_{jk}(0) = 2 \left( \psi^{1/2} + \psi^{-1/2} \right)^{-2},$$

where $\psi = (\rho_j c_{l,j}^{3/2} h_j^{5/2})/(\rho_k c_{l,k}^{3/2} h_k^{5/2})$, in which $\rho$ and $c_l$ denote the density and longitudinal velocity of the plate, respectively.

Different combinations of scaling parameters lead to different scaling forms [15]. In this paper, the thicknesses and the material properties are kept unchanged for simplicity. Suppose the lengths and widths of all SEA plate subsystems are scaled by a factor $\gamma_L$. It results in that $\tau_{jk}$ is invariant in the scaling process and

$$\gamma_l = \gamma_L \gamma_n^{-1} = \gamma_L^{-1}.$$  

Substituting Eq. (10) into Eq. (5) yields

$$\gamma = \gamma_L.$$
The modal density of a flat plate \( n \propto A/(hc_l) \), its scaling factor is
\[
\gamma_n = \gamma_A \gamma^{-1} \gamma_{c_l}^{-1} = \gamma_L^2.
\]
Substituting Eqs. (10) and (12) into Eq. (6) yields
\[
\gamma_P = \gamma_L.
\]
To reduce the computational cost in the transient response prediction, the modal density should be scaled down which results in an auxiliary requirement
\[
\gamma_L = \sqrt{\gamma_n} < 1.
\]
Equations (10), (11), (13), and (14) are the specific scaling forms for a plate assembly with line junctions.

4 Numerical validation

The specific scaling forms developed in Section 3 are applied to two identical coupled plates (1.0 x 0.7 m x 2 mm). The plates are made of aluminum, with material parameters: Young's modulus \( E = 71 \) GPa, density \( \rho = 2700 \) kg/m\(^3\), and Poisson's ratio \( \nu = 0.33 \). Rayleigh damping is considered and the damping ratio is 0.02 at 100 Hz and 5000 Hz. Both plates are simply supported and are coupled by their short edge at an angle of 120 degree, as shown in Figure 1.

![Figure 1: Two coupled simply supported plates](image-url)

Each plate has a wave length about 0.06 m at 5 kHz and a modal density of 111.6 modes per kHz. As a rule of thumb that one wave length is modeled by six linear elements, an FE model of about 7000 elements is built for each plate. A point load is applied on one plate at (x = 0.5...
m, y = 0.3 m). It is generated by filling a triangle waveform with a 5 kHz sinusoidal signal, as shown in Figure 2.

To speed up the computation, the coupled plates are scaled following the specific scaling forms. The control factor is chosen to be $\gamma = 1/2$. It results in a scaled model which has less than 27.9 modes per kHz. Each plate in the scaled model can be sufficiently described by 1750 linear plate elements. The damping ratio is doubled according to Eq. (10). The time variable is scaled to one half of its original value according to Eq. (11). Accordingly, the simulation time range for the scaled model is one half of the original value too. Since the driving point conductance of the plate remains unchanged and the power input is proportional to the force squared, one needs to scaled the magnitude of the point load by $\sqrt{2}/2$ in order to meet the requirement (13). The scaled load is shown in Figure 2. The averaged kinetic energy densities of the scaled plates are in good agreement with those obtained from the original system, as shown in Figures 3 and 4. The computation time using the scaled model drops to about one-eighth of that using the original system.

5 Conclusions

The paper presents a transient scaling approach to predict the high-frequency vibration of impulsively excited structures. The relationship of instantaneous power flow balance among subsystems is employed to derive the general scaling laws, which guarantee the dynamic similitude between the scaled model and the original system. Specific forms of the scaling laws are formulated for line-couple plates. Scalings are taken in the structure parameters and the time variable as well. Numerical validation demonstrates that a good response prediction in the time domain can be obtained from the scaled model. Since the damping is usually increased in the scaled model, the energy dissipates and transmits faster in the scaled model than in the original system. This property implies that one can simulate a short time range to get a broad picture of the evolution of the subsystem energies over a long one. Together with the
reduced model size, a small-sized FE model can be used to efficiently solve for the transient responses. In this sense, the present approach offers a way to extend the FE methods to the transient high-frequency vibration analysis.

Acknowledgements

This work was financially supported by the National Science Foundation of China (No. 11174041).

References


