Buenos Aires – 5 to 9 September, 2016
*Acoustics for the 21st Century…*

PROCEEDINGS of the 22nd International Congress on Acoustics

## Acoustic Array Systems: Paper ICA2016-731

# Spherical harmonic smoothing for DOA estimation of fully coherent sources

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## Abstract

The subspace-based estimation of direction of arrivals (DOAs) of multiple sound sources requires multiple observation data from a microphone array, such that basis of signal subspace can be fully identified. For coherent sources, however, multiple measurements only produce linearly dependent observations, which makes it difficult to separate signal subspace from noise subspace. In order to obtain linearly independent observations from fully coherent sources, various smoothing techniques including spatial, temporal, and frequency smoothing have been proposed. In principle, smoothing techniques acquire linearly independent observations by separating a single measurement into multiple subarray data. For constructing subarrays, however, several constraints on the array shape or waveform are need to be satisfied. In this work, we develop a smoothing technique based on spherical harmonic expansion that can be applied without any constraint on the array shape or waveform. The proposed technique directly produces linearly independent observations from spherical harmonic coefficients of measured data. Spherical harmonic coefficients of measured data are separated into multiple subsets in spherical harmonics domain, which correspond to linearly independent observations of fully coherent sources. The proposed smoothing technique is processed in the spherical harmonics domain, so it is compatible with various eigenbeam-based approaches such as EB-MUSIC and EB-ESPRIT, as far as spherical harmonic coefficients can be measured up to a finite order greater than one.

**Keywords:** Spherical harmonics, coherent source, DOA estimation, smoothing technique

# Spherical harmonic smoothing for DOA estimation of fully coherent sources

## 1 Introduction

Many directions of arrivals (DOAs) estimation techniques have several limitations in its practical applications. One particular problem that this paper focuses is the estimation of fully coherent sources, which refer to multiple sources strongly correlated to one another. For coherent sources, many subspace-based imaging techniques (e.g. Multiple Signal Classification (MUSIC) [1][2], Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [3]-[5]) fail to predict accurate DOAs due to lack of information on the signal subspace.

To circumvent problems associated with coherent sources, smoothing techniques that increase the number of independent observations on signal subspace have been proposed [6]-[8]. For spherical arrays, the spatial smoothing technique imposes a strong constraint on the way microphones are positioned: the entire array should be decomposed into subarrays of identical shape. There are other smoothing techniques (e.g. frequency smoothing [9], temporal smoothing [10]) that are not restricted by the microphone arrangement. Nonetheless, even for these techniques, there exist other requirements on spectral bandwidth or temporal sparsity. To overcome the restriction on the way array is arranged, smoothing techniques using different type of sensors have been investigated [11][12]. For example, polarization or vector field smoothing obtains independent observations using vector sensors polarized in different directions. However, the use of vector sensors requires extra measurement channels, and the identifiable number of coherent sources is limited by the number of vector sensors available.

The primary objective of this work is to develop a general smoothing technique that can be applied to general spherical arrays even in the single frequency case. Since most spherical beamforming or eigenbeam (EB) techniques [13][14] transform measured signals using spherical Fourier transform [15]-[18], it would be convenient if the smoothing can be accomplished only using the transformed spherical harmonic coefficients. The spherical harmonic smoothing technique proposed in this study directly increases subspace observations by constructing subarrays in spherical harmonics domain. As a result, the technique is compatible with many extant spherical beamforming techniques. The technique is derived for a single frequency case and can detect the location of each individual source under perfectly correlated condition.

### 1.1 Spherical microphone arrays

To begin with, consider the spherical harmonic expansion of multiple plane waves impinging on a spherical array of radius $a$. When $Q$ plane waves of complex amplitudes $s_q$ at a single frequency $\omega = ck$ (c: speed of sound, $k$ : wavenumber) propagate from directions $\Omega_q = (\theta_q, \phi_q)$ ( $q \in [1, Q]$), the total sound field measured by a microphone at $\Omega$ is given by

$$p(ka, \Omega) = \sum_{q=1}^{Q} g(ka, \Omega, \Omega_q) s_q + \eta(\Omega) , \tag{1}$$

where the microphones' self-noise is denoted by $\eta(\Omega)$, and assumed to be spatially uncorrelated white Gaussian with zero mean and variance $\sigma_V^2$. The propagating function of a single plane wave $g(ka,\Omega,\Omega_q)$ can be represented in terms of spherical harmonics $Y_n^m(\Omega)$ as [15][18]

$$g(ka,\Omega,\Omega_q)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n} b_n(ka)Y_n^m(\Omega_q)^* Y_n^m(\Omega), \quad ((\cdot)^*: \text{complex conjugate}) \tag{2}$$

where the spherical harmonics of order $n$ and degree $m$ is defined as [15][18]

$$Y_n^m(\Omega)=Y_n^m(\theta,\phi)=\sqrt{\tfrac{2n+1}{4\pi}\tfrac{(n-m)!}{(n+m)!}}\mathrm{P}_n^m(\cos\theta)e^{im\phi}, \tag{3}$$

from the associated Legendre functions $\mathrm{P}_n^m$ and azimuthal harmonics $e^{im\phi}$. The radial dependency $b_n(ka)$ can be described differently depending on the boundary condition of the array surface. For a rigid sphere, the radial dependency can be expressed as

$$b_n(ka)=4\pi i^n[j_n(ka)-j_n'(ka)h_n^{(2)}(ka)/h_n'^{(2)}(ka)], \tag{4}$$

where $j_n(ka)$ and $h_n^{(2)}(ka)$ denote the spherical Bessel function and the spherical Henkel function of the second kind, respectively. The prime $(\cdot)'$ indicates the first order derivative with respect to $ka$.

The contribution of a single spherical harmonics $Y_n^m(\Omega)$ in $g(ka,\Omega,\Omega_q)$, i.e., spherical harmonic coefficients $g_{nm}(ka,\Omega_q)$, can be derived from the spherical Fourier transform of $g(ka,\Omega,\Omega_q)$ over a unit sphere $S^2$. That is,

$$g_{nm}(ka,\Omega_q)=\int_{\Omega\in S^2} g(ka,\Omega,\Omega_q)Y_n^m(\Omega)^* d\Omega = b_n(ka)Y_n^m(\Omega_q)^*. \tag{5}$$

In practice, a sound field is captured by microphones at discrete locations $\Omega_\ell$ ($\ell\in[1,L]$), so the integral in Eq. (5) can be computed in various forms depending on microphone arrangements [18]. In this paper, we consider a general condition that spherical harmonic coefficients can be measured up to the finite order $N$. From Eqns. (1) and (5), the total sound field $p(ka,\Omega)$ can also be transformed into spherical harmonics domain. The result of the transform is given as

$$p_{nm}(ka)=\sum_{q=1}^{Q} g_{nm}(ka,\Omega_q)s_q+\eta_{nm}=\sum_{q=1}^{Q} b_n(ka)Y_n^m(\Omega_q)^* s_q+\eta_{nm}, \tag{6}$$

where $p_{nm}(ka)$ and $\eta_{nm}$ denote spherical harmonic coefficients of the total sound field and noise, respectively. In a matrix form, the spherical harmonic coefficients can be described in terms of a spherical harmonics matrix $\mathbf{Y}_Q\in\mathbb{C}^{Q\times(N+1)^2}$ as

$$\mathbf{p}_{nm}=\mathbf{G}_{nm}\mathbf{s}+\boldsymbol{\eta}_{nm}=\mathbf{B}\mathbf{Y}_Q^H\mathbf{s}+\boldsymbol{\eta}_{nm}, \quad ((\cdot)^H: \text{Hermitian transpose}), \tag{7}$$

where the spherical harmonic coefficients of the total field and self-noises are denoted by

$\mathbf{p}_{nm} = [p_{00}, p_{1,-1}, p_{10}, p_{11}, ..., p_{NN}]^T$ and $\boldsymbol{\eta}_{nm}$, respectively. The complex amplitudes of plane waves are denoted by a vector $\mathbf{s} = [s_1, ..., s_Q]^T$, and the spherical harmonic coefficients of plane waves are represented by $[\mathbf{G}_{nm}]_{(:,q)} = [g_{00}(ka, \Omega_q), g_{1,-1}(ka, \Omega_q), g_{1,0}(ka, \Omega_q), \cdots, g_{NN}(ka, \Omega_q)]^T$.

The diagonal matrix $\mathbf{B}$ has the radial function $b_n(ka)$ on its diagonal elements. Note that, in Eq. (7), the only frequency dependency is with its radial function $b_n(ka)$. Therefore, if we multiply $\mathbf{p}_{nm}$ by the inverse of $\mathbf{B}$, we can obtain a frequency independent vector $\bar{\mathbf{p}}_{nm}$. That is,

$$\bar{\mathbf{p}}_{nm} = \mathbf{B}^{-1}\mathbf{p}_{nm} = \bar{\mathbf{G}}_{nm}\mathbf{s} + \bar{\boldsymbol{\eta}}_{nm} = \mathbf{Y}_Q^H\mathbf{s} + \bar{\boldsymbol{\eta}}_{nm}. \tag{8}$$

Note that overbars are used to indicate the normalization by $\mathbf{B}$ ($\bar{\mathbf{G}}_{nm} = \mathbf{B}^{-1}\mathbf{G}_{nm}$, $\bar{\boldsymbol{\eta}}_{nm} = \mathbf{B}^{-1}\boldsymbol{\eta}_{nm}$). The normalized spherical harmonic coefficients $\bar{\mathbf{p}}_{nm}$ and the expectation operator $E$ can then be used to construct a covariance matrix $\bar{\mathbf{R}}$ in spherical harmonics domain.

$$\bar{\mathbf{R}}_{nm} = E\left[\bar{\mathbf{p}}_{nm}\bar{\mathbf{p}}_{nm}^H\right] = \mathbf{Y}_Q^H\mathbf{R}_s\mathbf{Y}_Q + \bar{\mathbf{R}}_{\eta,nm} \tag{9}$$

Accordingly, the total covariance matrix is the sum of the source covariance $\mathbf{R}_s = E[\mathbf{s}\mathbf{s}^H]$ transformed into spherical harmonics domain and the noise covariance $\bar{\mathbf{R}}_{\eta,nm} = E[\bar{\boldsymbol{\eta}}_{nm}\bar{\boldsymbol{\eta}}_{nm}^H]$. Under a noise-free condition, the rank of $\bar{\mathbf{R}}_{nm}$ depends on that of $\mathbf{R}_s$. For coherent sources, however, it is impossible to identify the subspace spanned by signals because the rank of $\mathbf{R}_s$ is one. To circumvent this rank deficiency problem, various smoothing techniques [6]-[12] have been proposed. Among them, we briefly review the polarization smoothing that is in close relation to the proposed technique.

## 1.2 Polarization smoothing

The polarization smoothing attempts to obtain linearly independent observations of coherent sources by using vector sensors. The rank of the source covariance matrix can be increased by having vector sensors measure three different particle velocity fields. In particular, the spatial derivatives of $Q$ noise-free plane waves at position $\mathbf{r}$ in Cartesian coordinates are given by,

$$\frac{\partial}{\partial x}p(ka,\Omega) = i\sum_{q=1}^{Q}k_{x_q}s_q e^{i\mathbf{k}_q\cdot\mathbf{r}},\ \frac{\partial}{\partial y}p(ka,\Omega) = i\sum_{\ell=1}^{Q}k_{y_q}s_q e^{i\mathbf{k}_q\cdot\mathbf{r}},\ \frac{\partial}{\partial z}p(ka,\Omega) = i\sum_{q=1}^{Q}k_{z_q}s_q e^{i\mathbf{k}_q\cdot\mathbf{r}}, \tag{10}$$

where the wavenumber of the $q^{\text{th}}$ plane wave is given by the vector $\mathbf{k}_q = (k_{x_q}, k_{y_q}, k_{z_q})$. Note that these spatial derivatives provide new observations of the source signal $s_q$ weighted by wavenumbers $k_{x_q}, k_{y_q}, k_{z_q}$. For the ease of derivation, we define normalized complex derivatives of the sound field as

$$p_{xy_\pm}(\Omega) = \tfrac{1}{ik}\left(\tfrac{\partial}{\partial x} \pm i\,\tfrac{\partial}{\partial y}\right) p(ka,\Omega) = \sum_{q=1}^{Q} D_q^{xy_\pm} s_q e^{i\mathbf{k}_q\cdot\mathbf{r}}, \tag{11}$$

$$p_z(\Omega) = \tfrac{1}{ik}\tfrac{\partial}{\partial z}\, p(ka,\Omega) = \sum_{q=1}^{Q} D_q^{z} s_q e^{i\mathbf{k}_q\cdot\mathbf{r}}. \tag{12}$$

The three spatial derivatives are normalized by $ik$. As a result, they are independent of the frequency. Again, the spatial derivatives yield the source signals $s_q$ weighted by three directional weights

$$D_q^{xy_+} = e^{i\phi_q}\sin\theta_q,\ D_q^{xy_-} = e^{-i\phi_q}\sin\theta_q,\ D_q^{z} = \cos\theta_q, \tag{13}$$

which depend on the propagating direction $\theta_q$ and $\phi_q$ of each plane wave. Because the propagating directions of plane waves are all different from one another, the weighted source signals become linearly independent of each other. Therefore, the measurement of normalized spatial derivatives of a given sound field can increase the rank of the source covariance matrix.

To be more precise, consider the sound field of $Q$ plane waves and its first order derivatives. The sound field of Eq. (1) measured by $L$ microphones can be written in a vector form as

$$\mathbf{p}^1 = \mathbf{GS1} + \boldsymbol{\eta}, \tag{14}$$

where $[\mathbf{p}^1]_{(\ell)} = p(ka,\Omega_\ell)$, $[\mathbf{G}]_{(\ell,q)} = g(ka,\Omega_\ell,\Omega_q)$ ,and $\boldsymbol{\eta} = [\eta(\Omega_1),\ldots,\eta(\Omega_\ell)]^T$. The matrix $[\mathbf{S}]_{(q,q')} = s_q\delta_{qq'}$ multiplied with a unit vector $\mathbf{1} = [1,\cdots,1]^T$ is equivalent to $\mathbf{s}$. Without self-noises $\boldsymbol{\eta}$, the normalized spatial derivatives of (14) can then be expressed as,

$$\mathbf{p}^\Delta = \mathbf{GSd}^\Delta, \tag{15}$$

where the superscript $(\cdot)^\Delta$ indicates the type of derivatives ($\Delta = \{1, xy_+, xy_-, z\}$), and the column vector $[\mathbf{d}^\Delta]_{(q)} = D_q^\Delta$ represents directional weights ($\mathbf{d}^1 = \mathbf{1}$ for a pressure field without derivative). The inspection of the rank of a horizontally concatenated matrix $\mathbf{P} = [\mathbf{p}\quad \mathbf{p}^{xy_+}\quad \mathbf{p}^{xy_-}\quad \mathbf{p}^z]$ allows us to confirm the linear independence of measured pressure fields. That is,

$$\mathbf{P} = \mathbf{GSD},\quad \text{where } \mathbf{D} = [\mathbf{1}\quad \mathbf{d}^{xy_+}\quad \mathbf{d}^{xy_-}\quad \mathbf{d}^z]. \tag{16}$$

The rank of the matrix $\mathbf{P}$ is increased to $\mathrm{Min}\,(4,\mathrm{Q})$, since the source signal matrix $\mathbf{S}$ has rank $Q$, and the columns of $\mathbf{D}$ are linearly independent of one another. Consequently, the rank of the smoothed covariance matrix $\tilde{\mathbf{R}} = \frac{1}{4}\sum_\Delta E\left[\mathbf{p}^\Delta\mathbf{p}^{\Delta H}\right]$ is $\mathrm{Min}\,(4,\mathrm{Q})$. Therefore, with the smoothed covariance matrix, various subspace-based DOA estimation techniques can be applied for fully coherent sources that are less than or equal to four.

The spatial derivative is a useful tool for circumventing the rank deficiency problem. Nonetheless, there is one major hurdle that prevents it from being implemented in practice: the use of different type of sensors (e.g. vector sensors) are necessary to capture the sound field. To circumvent this problem (i.e. the use of different sensors), we introduce a method that can directly extract the spatial derivatives of a given sound field without having to use vector sensors.

# 2 Spherical harmonic smoothing

## 2.1 Spherical harmonics and spatial derivatives

Suppose that a set of spherical harmonic coefficients $\bar{\mathbf{p}}_{nm}$ is identified from a given sound field. If the spatial derivatives of $\bar{\mathbf{p}}_{nm}$ can be obtained as well, we can increase the rank of a covariance matrix just like the polarization smoothing does. In order to derive the spatial derivatives only from the measured $\bar{\mathbf{p}}_{nm}$, we first need to understand how the spatial derivatives relate to the spherical harmonic coefficients. According to Eq. (15), the spatial derivatives only modify the source signal $\mathbf{S}$ through the derivative vector $\mathbf{d}^{\Delta}$. Therefore, the spherical Fourier transform of $\mathbf{p}^{\Delta}$ can be rewritten in terms of the modified source signal as follows:

$$\bar{\mathbf{p}}_{nm}^{\Delta} = \bar{\mathbf{G}}_{nm}\mathbf{S}\mathbf{d}^{\Delta} = \mathbf{Y}_Q^H\mathbf{S}\mathbf{d}^{\Delta}. \tag{17}$$

The remaining problem is to find out $\bar{\mathbf{p}}_{nm}^{\Delta}$ using the measured coefficients $\bar{\mathbf{p}}_{nm} = \mathbf{Y}_Q^H\mathbf{S}\mathbf{1}$. Because directions of plane waves and $\mathbf{d}^{\Delta}$ are unknown, it is impossible to calculate $\bar{\mathbf{p}}_{nm}^{\Delta}$ directly. However, one clue to solve this problem can be found from recurrence relations of spherical harmonics, which relate the propagating directions of plane waves $(\theta_q, \phi_q)$ to the shift in spherical harmonic coefficients. The recurrence relations for spherical harmonics are given by

$$\begin{aligned}
&\left(\sin\theta_q e^{i\phi_q}\right)Y_n^m(\Omega_q)^* = W_{n+1}^{m-1}Y_{n+1}^{m-1}(\Omega_q)^* - W_n^{-m}Y_{n-1}^{m-1}(\Omega_q)^* \\
&\left(\sin\theta_q e^{-i\phi_q}\right)Y_n^m(\Omega_q)^* = W_n^{m}Y_{n-1}^{m+1}(\Omega_q)^* - W_{n+1}^{-(m+1)}Y_{n+1}^{m+1}(\Omega_q)^*, \\
&\left(\cos\theta_q\right)Y_n^m(\Omega_q)^* = U_n^{m}Y_{n-1}^{m}(\Omega_q)^* + U_{n+1}^{m}Y_{n+1}^{m}(\Omega_q)^*
\end{aligned} \tag{18}$$

where coefficients $W_n^m$ and $U_n^m$ are defined as[17],

$$W_n^m = \begin{cases} \sqrt{\frac{(n-m-1)(n-m)}{(2n-1)(2n+1)}} & \text{for } n \geq |m| \\ 0 & \text{for } n < |m| \end{cases}, \quad U_n^m = \begin{cases} \sqrt{\frac{(n-m)(n+m)}{(2n-1)(2n+1)}} & \text{for } n \geq |m| \\ 0 & \text{for } n < |m| \end{cases}. \tag{19}$$

The left-hand sides of these recurrence relations imply that we can have three new spherical harmonic coefficients weighted by three different directional weights that are equal to $D_q^{xy_+}$, $D_q^{xy_-}$ and $D_q^z$ of the normalized spatial derivatives of plane waves (Eq. (13)), respectively. For $Q$ plane waves, the recurrence relations give the following vector relations:

$$\bar{\mathbf{p}}_{nm}^{xy_+} = \mathbf{W}_{n+1,m-1}\bar{\mathbf{p}}_{n+1,m-1} - \mathbf{V}_{n,m}\bar{\mathbf{p}}_{n-1,m-1}, \tag{20}$$

$$\bar{\mathbf{p}}_{nm}^{xy_-} = \mathbf{W}_{n,m}\bar{\mathbf{p}}_{n-1,m+1} - \mathbf{V}_{n+1,m+1}\bar{\mathbf{p}}_{n+1,m+1}, \tag{21}$$

$$\bar{\mathbf{p}}_{nm}^{z} = \mathbf{U}_{nm}\bar{\mathbf{p}}_{n-1,m} + \mathbf{U}_{n+1,m}\bar{\mathbf{p}}_{n+1,m}, \tag{22}$$

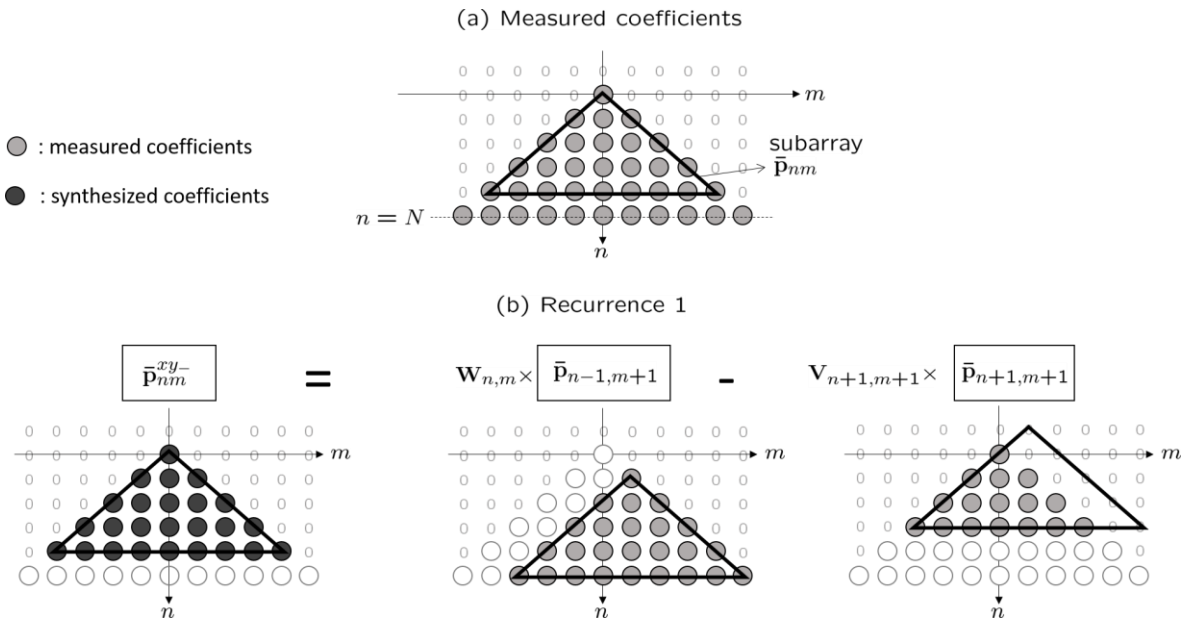


**Figure 1: Concept of spherical harmonic smoothing**
**(a) measured coefficients (b) rank enhancement using recurrence formula of Eq. (21)**

where $\bar{\mathbf{p}}_{n+\mu,m+\nu}$ represents the spherical harmonic coefficients $\bar{\mathbf{p}}_{nm}$ shifted by $\mu$ in order and $\nu$ in degree (Figure 1). Three diagonal matrices $\mathbf{W}_{nm}$, $\mathbf{V}_{nm}$ and $\mathbf{U}_{nm}$ express coefficients $W_n^m$, $W_n^{-m}$ and $U_n^m$ of Eq. (19), respectively.

Using Eq. (20)-(22), we can derive new observations of spherical harmonic coefficients ($\bar{\mathbf{p}}^{\Delta}_{nm}$) weighted by spatial derivatives, by using shifted sets of $\bar{\mathbf{p}}_{nm}$ in the spherical harmonics domain. However, a shifted set, e.g., $\bar{\mathbf{p}}_{n+1,m}$, requires higher order coefficients than those of the measured set $\bar{\mathbf{p}}_{nm}$. This problem can be resolved by defining $\bar{\mathbf{p}}_{nm}$ as an order-reduced subarray in the spherical harmonics domain (Figure 1) so that the shifted set is within the measured range of order and degree. For example, when the highest identifiable order of harmonics is given by $N$, $\bar{\mathbf{p}}_{nm}$ can be defined as a subset of coefficients up to the order $N-1$. Then all the shifted harmonics of Eq. (20)-(22) are within the range of measured harmonic coefficients.

The new observations ($\bar{\mathbf{p}}^{\Delta}_{nm}$) can be used to build a smoothed source-covariance matrix in spherical harmonics domain. That is,

$$\mathbf{R}_{nm} = E\left[\tfrac{1}{4}\sum\nolimits_{\Delta}\bar{\mathbf{p}}^{\Delta}_{nm}\bar{\mathbf{p}}^{\Delta\ H}_{nm}\right], \tag{23}$$

which can be utilized to many subspace-based DOA estimation techniques such as EB-MUSIC or EB-ESPRIT. Although we have explained the spherical harmonic smoothing only for spatial derivatives of the first order, formulas for higher-order derivatives can also be derived by combining recurrence formula of Eq. (18). The use of higher-order derivatives will increase the

number of identifiable sources, but in this case the order of a single subarray should be lowered, because calculation of higher-order derivatives requires a larger shift in spherical harmonics domain.

## 2.2 Performance evaluation

The performance of the proposed smoothing technique was investigated by applying the subspace-based DOA estimation techniques (EB-MUSIC and EB-ESPRIT) to the covariance matrix of Eq. (23). To simulate measurements with a spherical microphone array, $L = 32$ microphones were distributed over a rigid sphere of radius $ka = 2$ with their positions determined from the spherical t-design [16]. Spherical harmonics up to the order $N = 3$ are considered in simulations. It is assumed that plane wave sources produce monotonic (single frequency) sound and that the number of those plane wave sources is known a priori. Total 128 snapshots of temporal data were acquired to build the covariance matrix.

The first result, which is shown in Figure 2, compares EB-MUSIC spectrum with and without spherical harmonic smoothing. To evaluate the performance, four perfectly correlated plane wave sources with the equal power were positioned at $[\theta_1, \theta_2, \theta_3, \theta_4] = [20°, 46°, 140°, 169°]$, $[\phi_1, \phi_2, \phi_3, \phi_4] = [45°, 68°, 135°, 277°]$. White Gaussian noises of variance $\sigma_V^2$ were added to microphone signals to simulate microphone self-noises. Here, signal to noise ratio (SNR) is defined as the ratio of total power of plane waves to the sum of self-noise variances ($\mathrm{SNR(dB)} = 10\log_{10}(\sum_{q=1}^{Q} |s_q|^2 / L\sigma_V^2)$), which was set to 20dB for this simulation.

As shown in Figure 2 (a), the conventional EB-MUSIC fails to identify correct DOAs under fully coherent condition, because the rank of the source-covariance matrix is not sufficiently large. In contrast, EB-MUSIC with spherical harmonic smoothing can localize four source directions well, as shown in Figure 2 (b). Therefore, it can be concluded that the spherical harmonic smoothing can successfully de-correlate 4 fully coherent sources using the first order spatial derivatives.

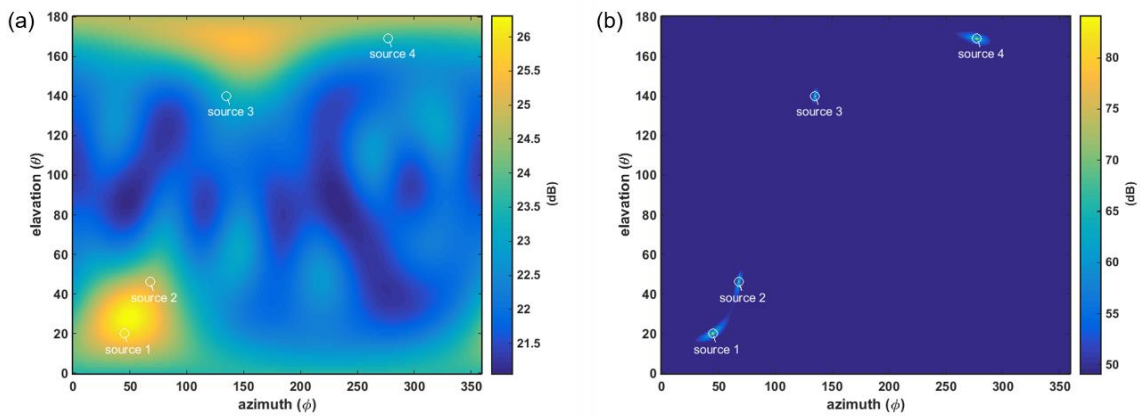


**Figure 2: EB-MUSIC spectrum with 4 fully coherent sources**
**(a) Conventional EB-MUSIC (b) EB-MUSIC with spherical harmonic smoothing**

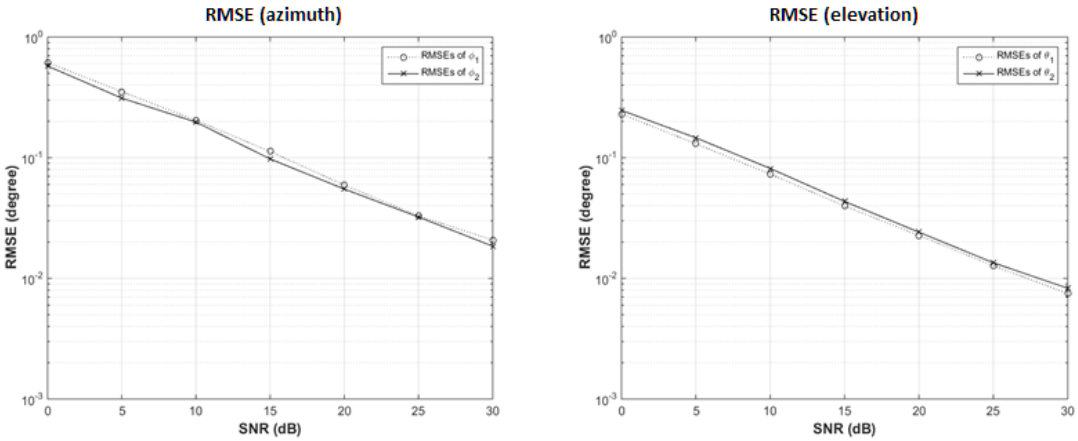


**Figure 3: RMSEs against SNR for spherical microphone array of 32 microphones**

For quantitative analysis of DOA estimation error, a parametric estimation technique (EB-ESPRIT) that directly estimates angles was applied to the smoothed covariance matrix. Two fully correlated plane wave sources with the equal power were configured at $(\theta_1, \phi_1) = (20°, 45°)$ , $(\theta_2, \phi_2) = (46°, 68°)$ . For various SNRs varying from 0dB to 30dB, estimation error for each source was collected over 400 trials and was averaged to obtain root mean squared error (RMSE). RMSEs for elevation and azimuthal angles are depicted in Figure 3 with respect to SNRs. RMSEs are inversely proportional to SNR for both sources, and the RMSEs remained quite small (under $1°$) regardless of the value of the SNR. This observation demonstrates that the proposed technique can localize fully coherent sources in combination with various eigenbeam-beamformers that take advantage of the orthogonality between signal plus noise subspace and noise subspace.

# 3 Summary

A new smoothing technique based on spherical harmonics is proposed for DOA estimation of fully coherent sources. The proposed technique increases the rank of signal subspace by using three recursion formula of spherical harmonics. The technique can be applied for any kinds of microphone arrays. In addition, it can be combined with other spherical beamforming techniques as far as the spherical harmonics can be extracted with minimal error. Simulation results with fully correlated sources demonstrate that the smoothing technique can localize multiple coherent sources when combined with EB-MUSIC and EB-ESPRIT. However, the aforementioned advantage of the new technique does not come without a cost: the order of spherical harmonic coefficients for a single subarray is inevitably reduced. Therefore, the proposed technique is suitable for subspace-based technique, which gains super-directivity from the orthogonality of subspaces, not by the absolute beamwidth of a beamformer.

## Acknowledgments

This research was supported by a grant from Agency for Defense Development under contract #UC150002ID, and the BK21 plus program through the National Research Foundation (NRF) funded by the Ministry of Education of Korea.

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