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Aeroacoustics of free reeds

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Abstract:

Free reeds, such as those found in the accordion and the harmonica, produce sound through complex flow-structure interaction. This study uses extensive experimental measurements of acoustic, aerodynamic and vibration phenomena to develop an improved physical understanding of how a free reed produces sound. We propose a new model for the instability of the reed and for how the oscillation of the reed tongue generates sound, examining how the characteristics of the sound change with the key parameters. Laser vibrometer and high speed camera measurements were used to examine the motion of free reeds. To characterise and distinguish the aeroacoustic sound sources, directivity measurements with far-field microphones were carried out, along with an inspection of the acoustic waveform's causal relationship to the position of the reed in its cycle. The experimental data matches well with simple theoretical modelling of the aeroacoustic sources. The key sources of sound were identified to be a dipole source due to the fluctuating force exerted on the fluid by the moving reed tongue, and a monopole source associated with the fluctuating mass flow of air through the reed slot. We show that the mass flow fluctuation is the dominant mechanism of sound radiation from free reeds.

Keywords: Free reeds

Aeroacoustics of free reeds

1 Introduction

The “Western” drawn-closed free reed, found for instance in the accordion, can be considered as a cantilever beam clamped over a similarly-sized slot in a plate. The reed vibrates when air is drawn through the slot at sufficient flow rates. By considering the mechanism for oscillation and the nature of the pulsating flow, this study explores the manner in which the oscillating system acts as a source of sound. The free reed produces sound at a clearly defined tone, with high harmonic content. The oscillating system is dominated by the characteristics of the reed tongue, with sound emitted at a frequency slightly below the unforced natural frequency of the reed’s first mode of vibration over a wide range of flow rates.

Figure 1: **A range of accordion reed plates, donated by Rees Wesson**

In order to understand the sources of sound in the free reed system a holistic study is necessary. It is crucial to understand the interaction between the flow of air past the reed and the vibration thereof, as these are the mechanisms by which sound is produced. In this study a new model for the sound generation by free reeds is presented, and validated against experimental data. The two key sources of tonal sound are identified as the fluctuating force exerted by the reed tongue on the flow and the oscillating mass flow past the reed. The fluctuating force acts as a “dipole” source and is dominant when the reed is plucked without airflow. The oscillating mass flow acts as an efficient “monopole” source of sound and is expected to dominate for the reed in flow.

2 Theory and Modelling

2.1 Flow Structure Interaction

When comparing laser vibrometer measurements of the reed in flow, with those from impulse hammer tests, it is shown that the reed vibrates predominantly in its first mode of vibration. The vibrating read tongue can be modelled as a clamped cantilever beam. The equation of motion for an Euler-Bernoulli beam is given by:

$$\rho A \frac{\partial^2 y}{\partial t^2} + B \frac{\partial y}{\partial t} + EI \frac{\partial^4 y}{\partial x^4} = f(x,t) \tag{1}$$

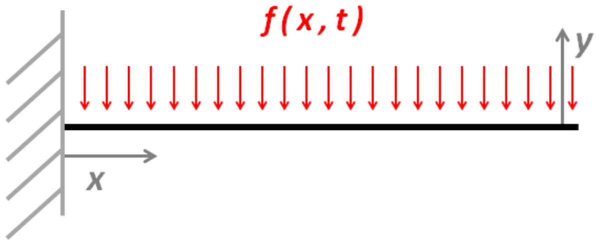

Figure 2: **Clamped-free cantilever beam with force per unit length $\mathbf{f(x,t)}$**

where y and x are as defined in Figure 2. A, ρ and EI are the cross-sectional area, density and flexural rigidity of the reed respectively. B is a term to capture viscous damping effects, and $f(x,t)$ is the transverse force per unit length exerted on the reed. To enable a rigorous treatment of damping effects, the equation of motion should be considered in the frequency domain. An experimentally determined Q factor (Q) can then be used to capture the damping from various mechanisms. The orthogonality property of mode shapes can be employed to account for the effect of a uniform pressure distribution acting as a modal forcing term. This is done by multiplying the frequency domain equation by the first mode shape, and then integrating along the length (l) of the reed with respect to x. The resulting equation of motion is:

$$\left(\frac{-\omega^2}{\omega_1^2}+\frac{i\omega}{\omega_1 Q_1}+1\right)\hat{y}=\frac{\beta_1}{\beta_2}\frac{1}{k_1^4 EI}f(\omega_{forcing}) \quad (2)$$

where k is the wavenumber, ω_1 is the natural frequency of the first mode and $f(\omega_{forcing})$ is the pressure force per unit length, which may be complex. Using mode shapes normalised to unit tip deflection, $\beta_1 = 0.3915l$ is the integral of the mode shape over the length of the reed and $\beta_2 = 0.25l$ is the integral of the mode shape squared over the length of the reed.

The pressure force acting on the reed can be determined by considering the flow past the reed. It is assumed that a jet is formed at the sharp edges of the reed and that the downstream surface of the reed is not wetted. The pumped flow is assumed to be negligible. The reduced velocities are sufficiently high to suggest a quasi-steady approach should be valid. This allows the flow past the reed to be calculated for a given reed position, based on a constant pressure difference between the upstream and downstream reservoirs ($p_1 - p_4$). The steady Bernoulli equation is applied across the reed, which shows that the pressure difference across the reed is equal to the dynamic pressure in the jet. The steady-flow momentum equation and incompressible continuity are employed to derive the pressure difference across the reed based on $p_1 - p_4$ and the instantaneous gap area (A_{gap}):

$$\Delta p_{2,3} = 0.5\rho_a V_j^2 = \frac{p_1 - p_4}{1-2\frac{\alpha A_{gap}}{A_4}(1-\frac{\alpha A_{gap}}{A_4})} \quad (3)$$

where all variables are as defined in Figure 3, ρ_a is the density of the air, p_i indicates the pressure at location i, and α is a jet contraction coefficient.

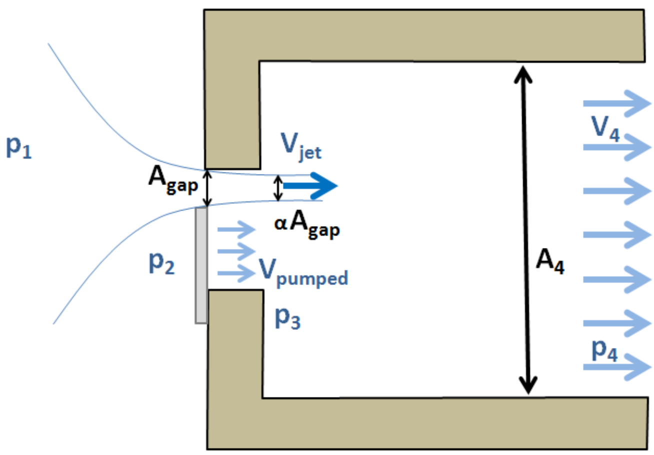

Figure 3: **Schematic of flow past the reed, with nomenclature used for the quasi-steady aerodynamic model**

Equation 3 gives a pressure force that is in phase with the displacement of the reed. In order to understand the mechanisms that feed energy into the system and allow self-sustained oscillation of the reed tongue, unsteady effects must be considered. To account for the unsteady effect of flow inertia, the unsteady Bernoulli equation should be considered for the air in a small channel of length δ behind the reed. To further improve the model, the effects of density fluctuations in the downstream volume can be considered using the compressible continuity equation. This analysis is explored in detail by Tarnopolsky [1] and Millot [2]. With modifications made to the flow-model to account for unsteady effects, it is possible to show that there is a component of the force that is in phase with the velocity of the reed, acting as a negative damping term by feeding energy into the oscillations. For this study, however, it will be assumed that, despite not capturing the mechanism for self sustained oscillation, the quasi-steady model gives a reasonable approximation of bulk trends in the flow. The quasi-steady model can thus be employed to calculate an estimate of the flow past the reed from the experimentally measured reed motion. The acoustic pressure emitted by the predicted sound sources is then calculated based on this estimation of the flow. The results are compared to acoustic measurements in order to examine the aeroacoustic hypothesis made in this paper.

2.2 Sources of Sound

For the free reed system the turbulence in the airflow will generate a non-tonal broadband sound with weak directionality. The shedding of large scale vortex structures, observed by Tarnopolsky using Schlieren imaging [3], will act as a sound source with a dipole directivity. However, as vortex shedding has been found to have no significant impact of the oscillation of the reed, the associated pressure fluctuations are likely to be small. Pulsation in mass flow is an efficient monopole sound source and is expected to contribute significantly to the radiated sound. A vibrating cantilever in free space would act as a dipole source, with pressure cancellation giving a strong directivity. For the framed reed the pressure cancellation will be less effective, especially when the reed is inside the slot, due to the geometric constraints. The radiated acoustic pressure is nevertheless expected to have certain dipole characteristics. This “dipole” sound source is modelled by considering the oscillating force exerted by the moving reed tongue on the surrounding air.

The rate of change of mass flux per unit volume ($\ddot{m}$) through the gap is modelled as a source term in the acoustic wave equation. By convolution with the free space Green’s function it is possible to derive an equation for the far-field pressure fluctuations as a function of $\ddot{m}$:

$$p'_m(\mathbf{r},t) = \frac{\ddot{m}(t - r/c_0)}{4\pi r} \tag{4}$$

Likewise, modelling the oscillating force distribution as a source term in the acoustic wave equation leads to:

$$p'_f(\mathbf{r},t) = \frac{\cos\theta}{4\pi c_0^2 r}\frac{\partial \mathrm{f}_1}{\partial t}(t - r/c_0) \tag{5}$$

where the total force exerted by the reed on the air is denoted by f, and c_0 is the speed of sound. $\mathbf{r}$ is the observer location at a distance of r from the reed and an angle of θ from the upstream perpendicular. The $\cos\theta$ term captures the dipole directivity of the unsteady force distribution, in contrast to the uniform directivity found for the mass flow source.

3 Experimental Techniques

In this study, directivity tests are carried out with an arc of microphones positioned in the far-field, as shown in Figure 4. Most previous experimental studies on free reeds have been conducted with a single microphone in the near-field [4], but this raises a number of issues as it is not possible to measure directivity patterns or distinguish between the acoustic and hydrodynamic field in the near-field.

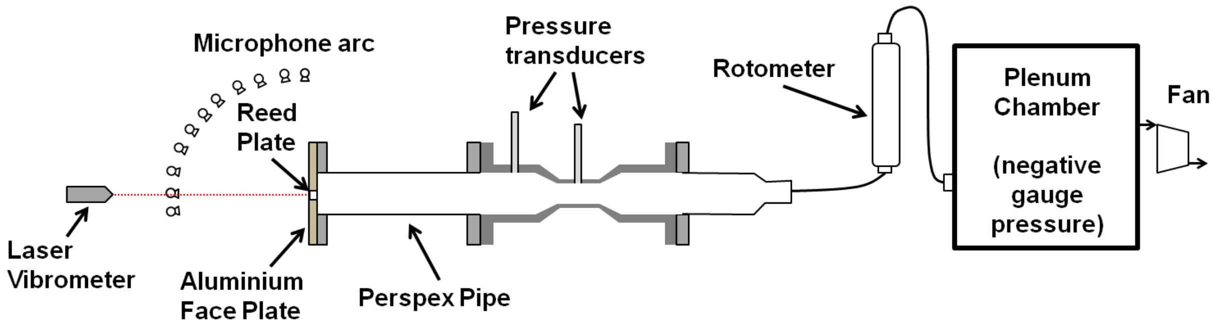

Figure 4: **Schematic of experimental rig**

The experimental setup consists of a suction supply, a perspex tube core with pressure instrumentation, a faceplate to hold the reed with aeroacoustic and vibrations instrumentation and electronic data acquisition kit sampled from a lab computer running MATLAB [5]. The suction supply is composed of a large reservoir with negative gauge pressure provided by a fan. The mean volume flow into the reservoir is measured by a rotometer. The main body of the rig is constructed from perspex pipe and includes a venturi tube equipped with pressure tappings that are used to measure the pressure fluctuations with *Kulite* pressure transducers.

The core of the rig is placed in an anechoic environment. The reed plate is slotted into a specially designed aluminium face plate. Plasticine used to seal the joints, avoiding rattle and air leakage. The aluminium face plate is bolted to the front of the perspex pipe, allowing air to be drawn past the reed by the negative gauge pressure in the reservoir. An arc of microphones is used to measure the far-field pressure distribution, and a laser vibrometer is used to measure the velocity of the reed.

The same setup is used to perform impulse hammer tests to characterise the modes of vibration. Simultaneous measurements with a laser vibrometer and a high-speed camera (HSC) are carried out to fully characterise the motion of the reed. Image analysis of the HSC frames allows the position of the reed with respect to the frame to be measured. It is important that this is done dynamically, rather than just measuring the static equilibrium position, because the equilibrium position of the reed varies with flow rate.

4 Results and Discussion

4.1 Modal Analysis

For the three reeds for which results are presented in this paper, the frequencies of the first transverse mode of vibration were found to be 236 Hz, 743 Hz and 633 Hz respectively. The modal quality factors were in the range of 200-400. The modal spacings between the different transverse modes observed differ significantly from those calculated by classical Euler-Bernoulli beam theory for a uniform reed. This discrepancy can be explained by the taper of the reeds used. It was found that, the stronger the taper, the greater the deviation from the ideal theory.

4.2 Time Domain Analysis

From the HSC image sequences it was found that oscillation initiates at a low amplitude, without reed closure, and then grows in amplitude to its steady value. As the flow through the reed is increased, the equilibrium position shifts downstream (towards the frame), and the proportion of the cycle for which the reed is within the slot increases.

The laser vibrometer velocity measurement is integrated to derive the reed deflection which is found to be sinusoidal. The image sequence from the HSC can be temporally aligned with the acoustic and vibrations measurements, allowing the points at which the reed enters the frame to be identified on the deflection waveform, as shown in Figure 5. The acoustic pressure fluctuations from a microphone at $\theta = 0^\circ$ are plotted at emitted time $(t - \frac{r}{c_0})$ to examine the causal relationship between the position of the reed and the acoustic pressure fluctuation radiated. Black vertical lines have been drawn through the points at which the reed passes through its equilibrium position, and where the reed enters the slot.

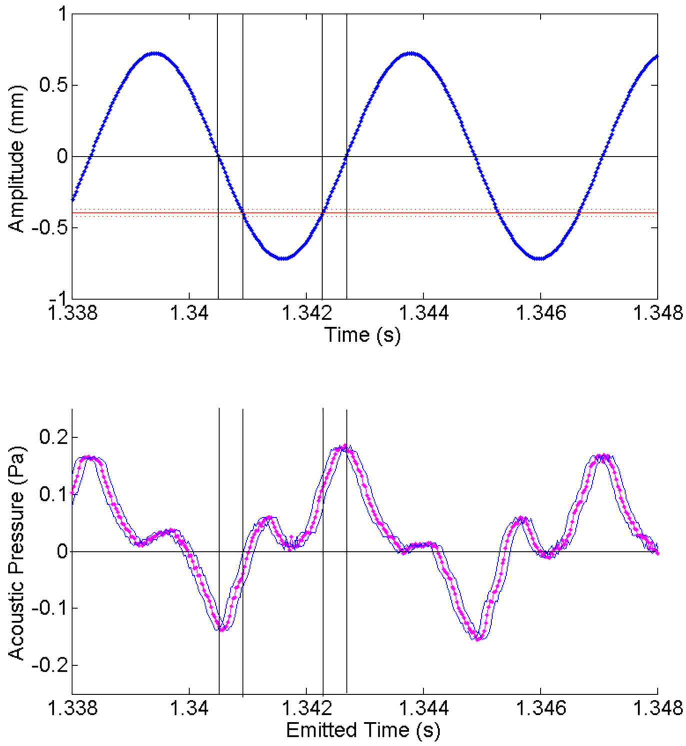

Figure 5: **Time domain measurements of Reed 1 with a mean flow rate of 21 l/min** (Top) **–•– reed deflection versus time, — deflection at reed closure, · · · uncertainty in reed closure,** (Bottom) **–•– acoustic pressure waveform at $\theta = 0^{\circ}$, — uncertainty in time shift to emitted time**

Figure 5 shows that the dominant peaks in acoustic pressure occur when the reed is moving through its equilibrium position, rather than when the reed is entering the slot - as has been claimed in the literature [4]. From Equations 4 and 5 it can be seen that the pressure fluctuations emitted by the two suggested sources of sound are in phase with the rate of change of flow through the reed, and the rate of change of the pressure force respectively. The quasi-steady model given in Equation 3 suggests that the rate of change of mass flow ($\ddot{m}$) has a negative peak as the reed passes through its equilibrium position while closing, as this is when the area through which the air can flow is decreasing most rapidly. Likewise there is a positive peak in $\ddot{m}$ as the reed moves through its equilibrium position while opening. The model further suggests that, while the reed is within the slot, the rate of change of the flow past the reed is low. This agrees very well with the key trends observed in the measured acoustic waveform, explaining the dominant peaks and plateaus. However, the application of the derived aeroacoustic source models to a quasi-steady flow does not explain some secondary features such the small peak in acoustic pressure just after reed closure. These additional features may be due to effects of the unsteady flow, rather than alternative sources of sound.

4.3 Harmonic Content

Figure 6 shows the variation in the amplitudes of harmonics with flow rate measured for the microphone positioned at $\theta = 0^\circ$ for Reed 1 and Reed 2. The harmonic content increases significantly with flow rate, especially the second harmonic which is very low at low flow rates.

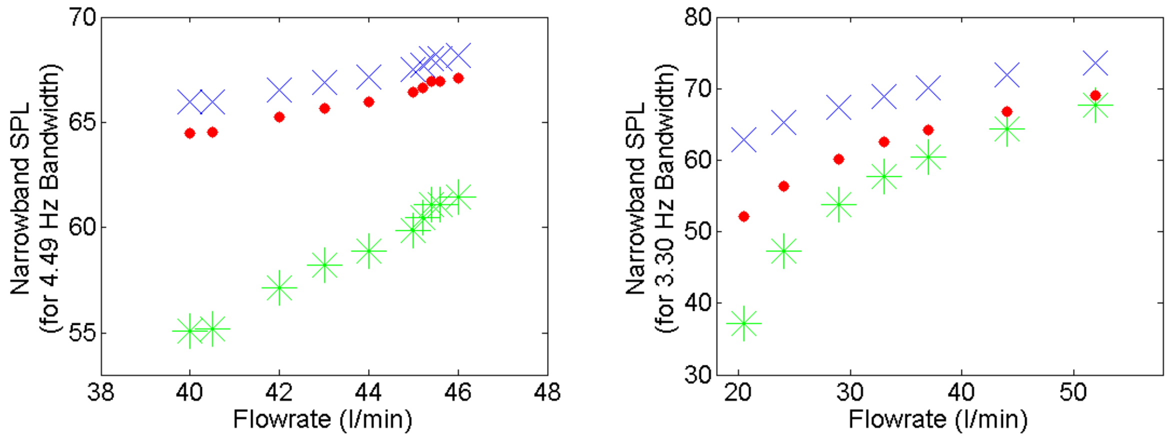

Figure 6: **Variation in amplitude of far-field acoustic pressure measured at θ = 0°, for fundamental and first two harmonics:** × ω_0 **,** ∗ $2\omega_0$ **,** • $3\omega_0$**.** (Left) **Reed 1,** (Right) **Reed 2**

These observations can be explained by considering how an increase in flow rate will affect the two key sources of sound identified. Equating the steady terms in Equation 2 shows that, as the flow rate increases, the equilibrium position of the reed will shift downstream. The higher the flow rate, the closer the equilibrium position is to the frame, and thus the less time there is between the peaks in acoustic pressure and the region of low acoustic pressure when the reed is closed. An increased flow rate thus leads to a "spikier" acoustic pressure waveform from both sound sources. This suggests that, if the aeroacoustic hypothesis presented in this paper is valid, there should be an increase in harmonic content (especially of the second harmonic) with flow rate, which is clearly the case.

4.4 Directivity Analysis

The final piece of evidence to support the hypothesis of the oscillating-mass-flow and the oscillating-pressure-force as the two key sources of sound comes from directivity measurements. Figure 7 shows the directivity of the first three harmonics measured for Reed 1, at two different flow rates.

The uncertainty in Sound Pressure Level (SPL), taking into account both the quoted microphone accuracy and the positioning of the microphones is 1.6 dB_{SPL}. The fundamental component has negligible directivity which suggests that it is dominated by a monopole sound source. The first and second harmonics have greater directivity, with some dipole characteristics.

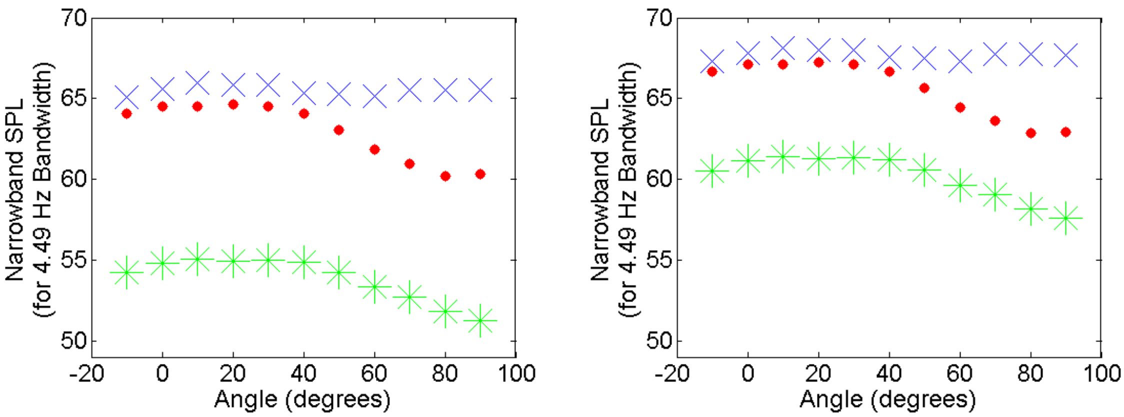

Figure 7: **Directivity plot for Reed 1, for fundamental and first two harmonics:** × ω_0 , * $2\omega_0$, • $3\omega_0$. (left) **Mean flow rate 40 l/min,** (right) **mean flow rate 46 l/min**

Considering Equations 4 and 5, with a quasi-steady model for $\ddot{m}$ and $\dot{f}$, the pressure fluctuations from the two sources should be in phase. This allows a decomposition of the directivity field into a monopole and a dipole component. Carrying out such a decomposition shows that the monopole component of the fundamental dominates the sound emitted for all reeds tested. The dipole component becomes more significant for higher harmonics. This can be explained by considering that the fluctuating-force (dipole) term varies as $\frac{\partial}{\partial t}(q_r/A_{gap})^2$ while the fluctuating-mass-flow (monopole) term varies as $\frac{\partial}{\partial t}q_r$. The flow area ($A_{gap}$) and the volume flow rate past the reed q_r are both non-linear. The fluctuating-force term is thus clearly more non-linear than the fluctuating-mass-flow term, resulting in higher harmonic content. The directivity results thus support the model of a dominant monopole source due to the fluctuating flow of air past the reed, accompanied by a weaker dipole source from the unsteady pressure force exerted by the moving reed on the flow.

5 Conclusions

The free reed system can be analysed effectively by modelling the reed tongue as a clamped cantilever beam subject to aerodynamic forcing from the flow past the reed. A quasi-steady model for this flow can be used to show how the suggested aeroacoustic sources can explain some of the key features in the sound emitted by free reeds.

The dominant source of sound from the free reed system is suggested to be the pulsation of the airflow past the reed. It is shown from a quasi-steady analysis that the rate of change of mass flow peaks when the reed is at its equilibrium position, and is low for the period during which the reed moves through the frame. At higher flow rates, the equilibrium position will move closer to the frame, so the time between the peak in $\ddot{m}$, and the region for which $\ddot{m}$ is very low decreases. This results in a sharper peak and higher harmonic content. An equivalent result is found for $\dot{f}$.

The temporal alignment of the measured acoustic pressure waveform with the reed deflection gives an excellent match to the expected result for a dominant pulsating-mass-flow sound source (applied to a quasi-steady model of the flow). The real pressure waveform exhibits some additional features which the quasi-steady model fails to capture.

The explanation provided for the dominant peaks also provides a feasible explanation for the rise in harmonic content with flow rate that is observed experimentally across all the reeds tested. A more general explanation for the high harmonic content is the non-linear variation of the flow area (A_{gap}) with reed deflection. The above described effect is one aspect of this non-linearity.

Inspection of the far-field acoustic pressure directivity shows a non-negligible dipole component, especially for higher harmonics. This dipole term increases in significance compared to the monopole term for higher harmonics, and may even exceed the monopole term in some cases. It is suggested that this is due to the fact that the pressure force across the reed is influenced more heavily by the non-linearities in the flow than the flow rate of air. Despite this non-negligible dipole term, attributed to the oscillating force from the reed on the surrounding air, the sound produced by all reeds tested was dominated by the monopole component of the fundamental. The supports the hypothesis presented in this paper that the sound produced by the free reed is dominated by the sound source due to the pulsating flow of air past the reed.

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References

[1] Tarnopolsky, A. Z. Fletcher, N. H. Lai, J. C. S. Oscillating reed valves - An experimental study. *Journal Acoustical Society of America*, Vol 108 (1), (2000), pp 400-406.

[2] Millot, L. Cuesta, Ch. Valette, C. A Proposal for a Minimal Model of Free Reeds. *Acta Acustica United with Acustica*, Vol 93, 2007, pp 122-144.

[3] Tarnopolsky, A. Z. Lai, J. C. S. Fletcher, N. H. Flow structures generated by pressure-controlled self-oscillating reed valves, *Journal of Sound and Vibration*, Vol 247 (2), 2001, pp 213-226.

[4] Cottingham, J. P. Reed Vibration in Western Free-Reed Instruments *Proceedings of 19th International Congress on Acoustics*, Madrid, 2007, Online.

[5] MATLAB *R2012b*, The MathWorks Inc. Massachusetts, 2012.