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RBF-based shapes optimized with genetic algorithms for sound diffusion

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Abstract

Development of sound diffusion technical solutions has been a topic of intense research in the last years. Many diffuser shapes, based on mathematical series or in different optimization techniques, have been suggested. In this paper, the authors propose an alternative technique to define new shapes of sound diffusion configurations, based on the use of radial basis functions (RBF). In addition, to allow the definition of optimal surface shapes for a given frequency band, a genetic algorithm is used. The diffusion coefficient is computed within the optimization procedure using the Kirchoff integral equation. The global procedure presented here allows simple organic shapes to be obtained, which are of significant interest in architectural design of acoustic spaces.

Keywords: Sound Diffusion; RBF; Genetic Algorithms; BEM.

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1 Introduction

Sound diffusers are a common technical solution used in the last four decades for conditioning performance rooms with greater acoustic requirements, such as theatres, concert halls or auditoria. They are applied to enrich the sound field in the performance spaces without presenting too much sound absorption, while scattering the sound energy uniformly around the room [1]. In order to achieve proper sound diffusion, the surfaces of the room can be shaped, surface ornamentation can be used and/or specific elements can be adopted, on the walls and ceilings, like sound diffusers.

A significant number of the acoustic diffusers commercially available are based on the phase grating diffusers or Schroeder-type diffusers. These are obtained by a series of adjacent wells of the same width, separated by thin walls, whose depths can be defined by a (simple) mathematical number sequence, e.g. a sequence of quadratic residues, being known asquadratic residue diffusers (QRD), among other Schroeder diffusers. However, in some particular cases, the visual appearance of the acoustic conditioning of the room with QRDs is considered by architects to be unaesthetic or visually unattractive in modern spaces [1], and thus other geometrical forms of the diffusive surfaces or elements need to be customized and explored.

The optimization of the diffusers' design has not been addressed by many researchers. Cox [1, 2, 3] describes the optimization processes for achieving improved performance stepped and curved acoustic diffusers. A predictive scattering model is complementing an optimization algorithm in the (iterative) search for the minimum error between the predicted and the desired scattering from the diffusive designed elements or curved surface. Perry [4] adopted a similar approach and used a finite difference time domain (FDTD) prediction model and an evolutionary optimization algorithm to iteratively design stepped diffusers or fractals based on stepped diffusers. More recently, Redondo et al [5] explored the optimization of sonic crystals to act as acoustic diffusers over a large frequency range. They used optimization techniques, namely evolutionary multiobjective algorithms, combined with an FDTD scheme to predict and considerably enhance the performance of the sound diffusers based on sonic crystals.

In the present work, simple organic (curvilinear) shapes are defined, based on the use of radial basis functions (RBF), and optimized towards the diffusive behaviour and aesthetic requirements. Next, the proposed mathematical formulationis described, including the numerical strategy for analysing the diffuser, the definition of its geometry and the shape optimization procedure. Then, a set of application results are presented and discussed.

2 Mathematical formulation

Sound diffuser performance is usually quantified by means of the Sound Diffusion Coefficient, which gives an idea of the capacity of a diffusing device to spread sound energy in space. This parameter is evaluated from the polar scattering diagram of a given diffuser configuration, by means of the equation:

$$d_\theta = \frac{\left(\sum_{i=1}^{n} 10^{L_i/10}\right)^2 - \sum_{i=1}^{n}\left(10^{L_i/10}\right)^2}{(n-1)\sum_{i=1}^{n}\left(10^{L_i/10}\right)^2} \tag{1}$$

where θ is the incidence angle of the sound waves, L_i is the Sound Pressure Level (SPL, in dB) at receiver position i, obtained from the polar response, and n is the number of receiver points used for the evaluation of the polar response.

In this paper, the analysis of the sound diffusers is performed numerically, and so the SPL at different receiver positions is addressed using well established numerical tools. The analysis procedure and the strategy defined for shape optimization of the diffusers is briefly described in the next subsections.

2.1 Numerical method

The Boundary Element Method (BEM) is usually considered one of the best tools for computational diffuser analysis, since it allows describing the geometry of a given problem just by discretizing the boundary surfaces of any objects that exist in the propagation medium. Considering that the sound propagation can be described by the Helmholtz equation, and that a 2D analysis can reproduce the sound field, it is possible to state that if an excitation source is located at $\underline{x}_F = (x_F, y_F)$, oscillating with an angular frequency ω, the corresponding incident field can be given by:

$$p^{inc}(\underline{x}, \underline{x}_F, \omega) = \frac{-\mathrm{i}A}{2} H_0^{(2)}\left(k_\alpha \sqrt{(x-x_F)^2 + (y-y_F)^2}\right) \tag{2}$$

where inc represents the incident field, A is the amplitude, α the propagation velocity, and $\mathrm{i} = \sqrt{-1}$. In addition, the wave number can be defined as $k_\alpha = \omega/\alpha$, and $H_n^2(...)$is the Hankel's function of the second kind and order n.

In the presence of a rigid, perfectly reflecting surface, the reflected field can be computed using a direct BEM formulation. Assuming that Γ is the diffuser's interface, the following integral equation can be written:

$$c \cdot p(\underline{x}_0, \omega) = -\int_{\Gamma_2} H(\underline{x}, \nu_n, \underline{x}_0, \omega) \cdot p(\underline{x}, \omega)\, d\Gamma + p^{inc}(\underline{x}_0, \underline{x}_F, \omega) \tag{3}$$

where H is the first derivative of the Green's function with respect to the outwards pointing normal to the surface (ν_n), considering a load acting at boundary point $\underline{x}_0$; c depends on surface's geometry and has a value of $1/2$ if $\underline{x}_0 \in \Gamma$ and for a smooth boundary, and 1 for a point in the domain. Solving this equation allows determining the pressure along the diffuser's boundary.

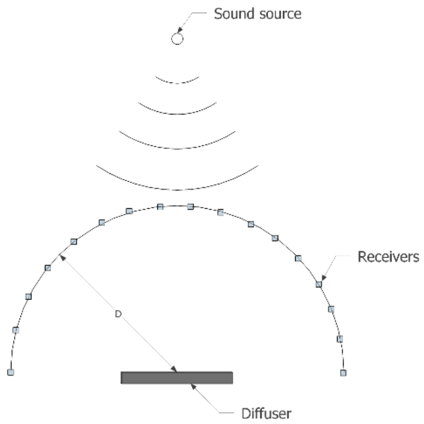


Figure 1 – Scheme representing the typical setup for diffuser analysis.

Although this is a well-established method for the analysis of diffusers, it can still be quite time-consuming to use if an optimization process (where hundreds of configurations may be analyzed) is to be implemented, as is the case in the present paper. In those cases, a simplified strategy, corresponding to Kirchoff's approximation, may be used. In that case, no equation system is assembled, and the pressure at each element is assumed to be $p(\underline{x},\omega)=2p^{inc}(\underline{x},\underline{x}_F,\omega)$. In practice, this is only valid if the interaction between surfaces is neglected, and is accepted if the analysis is not performed in the low-frequency range. Given this simplification, the response at any point in space in the presence of a diffuser can be written as:

$$p(\underline{x}_0,\omega) = -2\int_{\Gamma_2} H(\underline{x},\nu_n,\underline{x}_0,\omega)\cdot p^{inc}(\underline{x},\underline{x}_F,\omega)\,d\Gamma + p^{inc}(\underline{x}_0,\underline{x}_F,\omega) \quad (4)$$

This is, in practice, quite simple to implement, and allows for a very fast analysis of different configurations.

2.2 Definition of the geometry

The typical diffuser shape to analyze in the present paper corresponds to a smooth organic shape, based on a number of control points, through which a smooth mathematical curve is defined. To define this organic shape, an interpolation scheme based on the use of Radial Basis Functions (RBF) is used, and the MQ RBF (Multi-Quadric) has been chosen. This type of function is mathematically simple, depending only on the distance and on a free parameter, and is defined as:

$$\phi(\underline{x}) = \sqrt{r^2 + c^2} \quad (5)$$

Considering a number NC of control points, with coordinates (x^i, y^i), a possible interpolation scheme can be assembled using a set of NC RBFs, each one centered at one control point, such that:

$$\sum_{j=1}^{NC} A_j \phi_j (x^i) = y^i, \text{ for each } i = 1...NC \quad (6)$$

Applying Equation (6) to each collocation point, a system of NC equations on NC unknowns is generated, and its solution allows obtaining the amplitudes A_j of each RBF. A schematic representation of the obtained interpolating curve is depicted in Figure 2, for 10 control points. It should be noted that, in the present study, and as illustrated in the example of Figure 2, the definition of the diffuser shape is performed considering a pre-defined number of control points, equally spaced through a fixed width, and which have only two possible y coordinate values, $y = 0$ or $y = y_{\max}$ ($y_{\max}$ being a user-specified value).

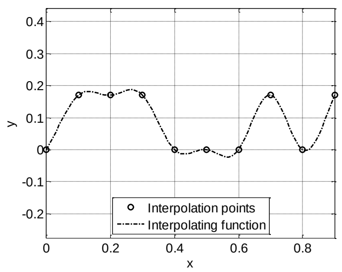


Figure 2 – Example of an interpolation curve computed for 10 control points, generating apossible smooth surface.

Since, in this paper, the acoustic analysis is performed using a boundary method (using Kirchoff's approximation), the generated organic shape is then discretized in a number of straight segments, always ensuring that a minimum of 10 segments per wavelength is used.

2.3 Shape optimization

Given the above formulation and mathematical details, it is now important to define all the optimization procedure used to define the optimized organic diffusive surfaces. This optimization is based on the use of a Genetic Algorithm, and is described in a simplified manner in the flowchart of Figure 3.

At the end of this process, a final organic (smooth) shape is obtained, defined in terms of RBF superposition, with optimal performance for the selected frequency band.

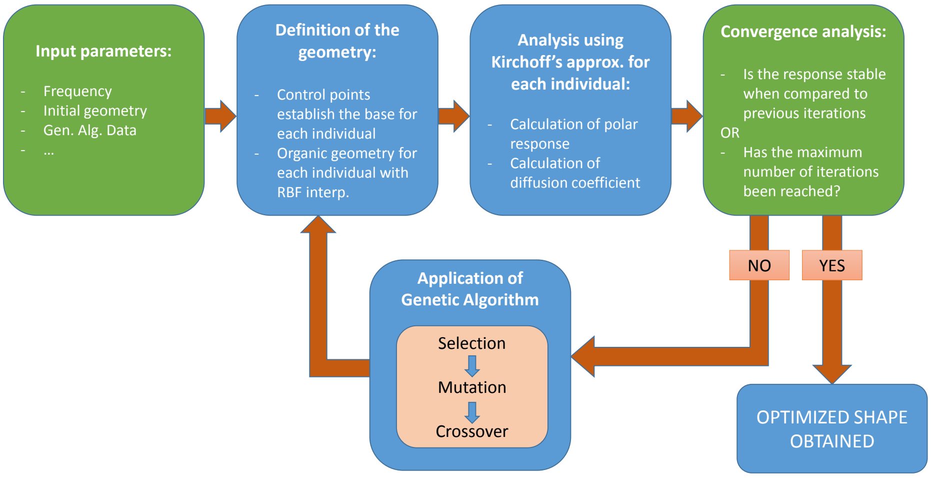


Figure 3 – Flowchart of the calculation/optimization process.

3 Case studies

3.1 Input parameters

As described in the previous section the input data are: the frequency (octave) band to optimize, ***f***; the number of frequencies within the octave band used to calculate the diffusion coefficient, ***nfreq_bands***; the dimension of the sound diffuser (in this case it was 1.90m); the number of the control points, ***controlpoint***, for the RBF (also defines the horizontal spacing between them, ***refh***, since the width of the diffuser is fixed) and the height of each control point, ***refv***. For all the test cases, the initial configuration was a flat plate (*refv*=0.0 for all points).

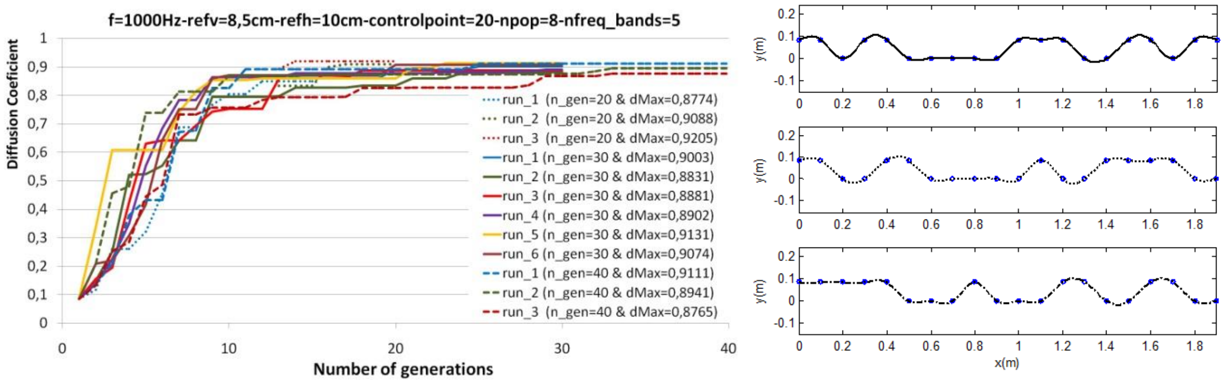
For compactness, the genetic algorithm control parameters (selection, crossover and mutation) will not be addressed in detail, and remain constant for all examples. At this stage the aim was to evaluate only the influence of the number of individuals of a population, ***npop***, and the number of generations, ***ngen***, defining the number of iterations.

For the calculation of the diffusion coefficient, ***d***, according to Figure 1, the sound source is positioned at 0° (normal incidence) facing the midpoint of the diffuser, at a distance of 500m. 180 receivers are arranged in an arc of a circle with r=250m, centered with the diffuser.

3.2 Convergence

The first analysis was on the convergence of the optimization for a local maximum (maximum diffusion coefficient in the frequency band to optimize, d_{Max}). For this, several consecutive tests were conducted without changing any input parameter, and then checking the final configuration (RBF) and the maximum diffusion coefficient obtained.

As can be seen in Figure 4a), the optimization process exhibits some variability, and does not lead to a unique maximum value, although the dispersion of the values is small (4% relative to the mean). This behavior may be better controlled by changing parameters that control GA (selection, mutation and crossover).



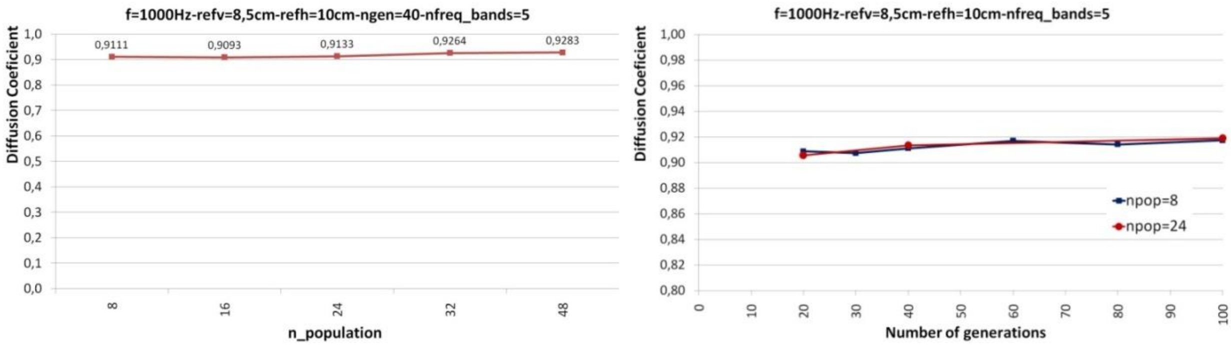
4a) 4b)

Figure 4: a) Several test runs (same input data); b) RBF configurations for the 3 test runs ngen=40.

In Figure 4b) the optimized configurations are shown for the test runs, after 40 generations. Although these configurations are different, they exhibit very similar shapes, which apparently correspond to lateral shift deviations and/or inversion.

3.3 Size of the population, *npop*

In order to test the influence of the size of the population, several test runs were performed. Some results are presented in Figure 5. It can be said that the influence of the number of individuals (***npop***) is not significant (although $\boldsymbol{d_{max}}$ slightly increases with the increasing number of individuals). From Figure 5b) we can say that the variation of the $\boldsymbol{d_{max}}$ obtained is small, even when the number of generations varies (***ngen***).

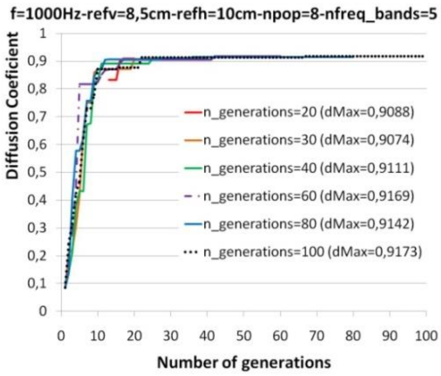


5a)Number of generations fixed (=40) 5b) Number of generations vary

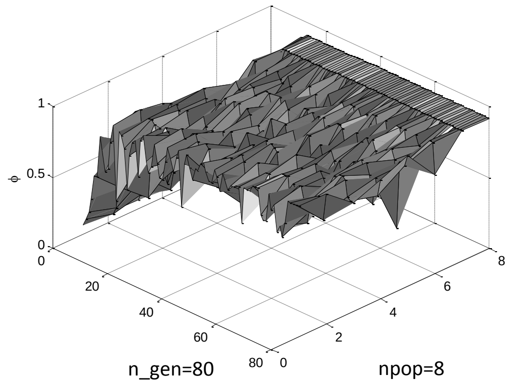
Figure 5: The influence of the size of the population.

3.4 Number of generations, *ngen*

Figure 6 shows the results for some test runs in order to observe the influence of the number of generations (number of iterations). From this figure, it can be inferred that 40 generations are sufficient (for the conditions indicated in the figure) to obtain the optimal or quasi-optimal solutions ($\boldsymbol{d_{max}}$). From Figure 6b) it is possible to say that the variation of the $\boldsymbol{d_{max}}$ obtained is small, even when the size of the population (***npop***) is small.



6a) Evolution of d_{max}



6b) Evolution of the population

Figure 6: The influence of the number of generations.

3.5 Height of the control points, *refv*

The height of the control points is an important geometric parameter of the configuration. To show the importance of ***refv***, Figure 7 illustrates the variation of d_{max} with the geometrical height, when the optimization is performed for 1000Hz. Clearly, if the height is too small when compared with the wavelength (0.34m), the diffuser will have a poor performance; when ***refv*** reaches ¼ of the wavelength (0.085m) excellent diffusion is obtained, with d_{max} reaching values around 0.9. It is interesting to note that a new decrease occurs when the height is half a wavelength, after which the performance seems to stabilize. Clearly, this can be attributed to the constructive/destructive interaction between the various reflections.

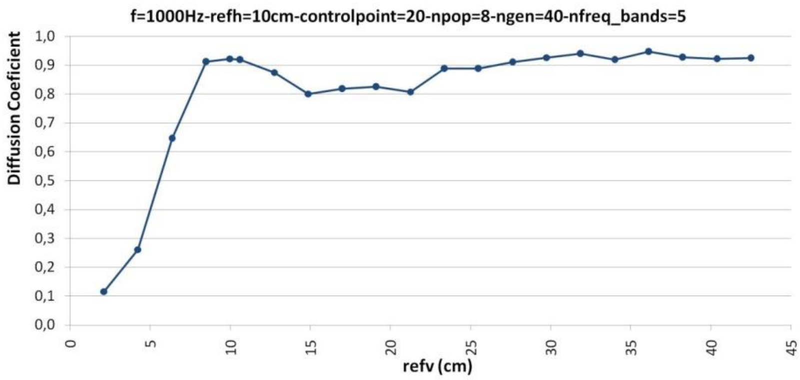


Figure 7: The influence of the height of the control points, *refv*.

3.6 Number of control points

Since the diffuser size is fixed (1.90m), varying the number of control points will lead to smaller spacing between them, and thus to sharper geometry variations. From Figure 8 (computed for 1000Hz) it can be seen that a minimum of 15 points (spaced 0.135m) should be used to define a sufficiently irregular surface in order to induce a diffuse reflection. As higher frequencies are considered, more and more control points should be used, in order to originate irregularities compatible with the wavelength.

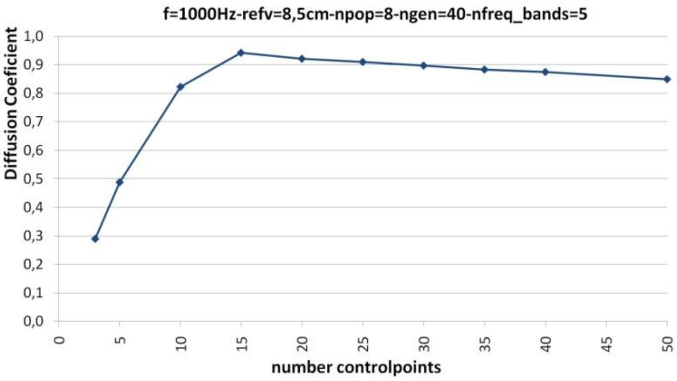
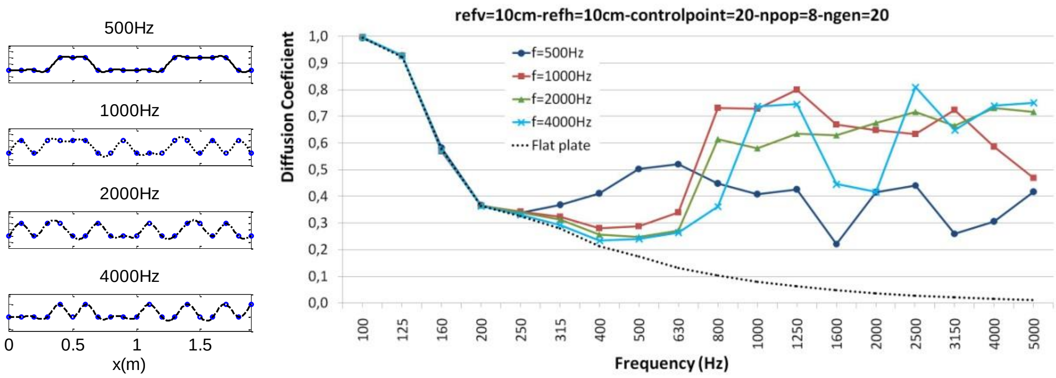


Figure 8: The influence of the number of control points.

3.7 Sound diffusion coefficient

For some optimized shapes, Fig. 9a), we calculated the sound diffusion coefficient, Fig. 9b), along the frequency range.



9a) Optimized shapes   9b) Sound diffusion coefficient of optimized shapes

Figure 9: Optimized diffusers.

As expected, the optimization for the lower frequency leads to stepped diffusers and for higher frequencies we obtain more irregular surfaces.

The analysis of the diffusion coefficient curve, we can generally state that the diffusers have a good diffusion in the respective frequency bands being the best in their respective octave band. The worse case is for the one optimized for the higher frequency but the spacing between the control points aren´t optimized for a such high frequency.

The maximum values are not identical to those obtained in the optimization algorithm and not in exactly the specific frequency because for the computation of the graph of Fig. 9b) 5 frequencies were used for each third octave band and on the optimization algorithm 5 frequency were used by octave band.

4 Conclusions

In the present work, the simple curvilinear shapes are defined, based on the use of RBF and its shape optimization gave us good results and demonstrated that this is a possible way to optimize the diffuser surfaces. The computational effort used is not very heavy and the optimization is quite rapid, which makes the numerical procedure presented here, although still at an embryonic stage of development, an extremely useful tool in the development tuned diffusing surfaces.

The next steps are tuning parameters that control the GA (selection, mutation and crossover) to obtain a lower dispersion results (towards the “unique solution”, if exists!) and allowing the variation of the geometric coordinates of the control points of the RBF (in particular, to have variable height) and apply geometric constraints to extreme control points to reproduce actual situations of hypothetical manufacturing.

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